



Ministry of Education, Science and Technology

# Lesson plans for JJSJ Mathematics

JSS **3** 

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#### Foreword

Our country's future lies in the education of our children. The Government of Sierra Leone is committed to doing whatever it takes to secure this future.

As Minister of Education, Science and Technology since 2007, I have worked every day to improve our country's education. We have faced challenges, not least the Ebola epidemic which as we all know hit our sector hard. The Government's response to this crisis – led by our President – showed first-hand how we acted decisively in the face of those challenges, to make things better than they were in the first place.

One great success in our response was the publication of the Accelerated Teaching Syllabi in August 2015. This gave teachers the tools they needed to make up for lost time whilst ensuring pupils received an adequate level of knowledge across each part of the curriculum. The Accelerated Teaching syllabi also provided the pedagogical resource and impetus for the successful national radio and TV teaching programs during the Ebola epidemic.

It is now time to build on this success. I am pleased to issue new lesson plans across all primary and JSS school grades in Language Arts and Mathematics. These plans give teachers the support they need to cover each element of the national curriculum. In total, we are producing 2,700 lesson plans – one for each lesson, in each term, in each year for each class. This is a remarkable achievement in a matter of months.

These plans have been written by experienced Sierra Leonean educators together with international experts. They have been reviewed by officials of my Ministry to ensure they meet the specific needs of the Sierra Leonean population. They provide step-by-step guidance for each learning outcome, using a range of recognised techniques to deliver the best teaching.

I call on all teachers and heads of schools across the country to make best use of these materials. We are supporting our teachers through a detailed training programme designed specifically for these new plans. It is really important that these Lesson Plans are used, together with any other materials you may have.

This is just the start of education transformation in Sierra Leone. I am committed to continue to strive for the changes that will make our country stronger.

I want to thank our partners for their continued support. Finally, I also want to thank you – the teachers of our country – for your hard work in securing our future.

Dr. Minkailu Bah

Minister of Education, Science and Technology

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## Introduction to the Lesson Plan Manual

## JSS **Mathematics**

These lesson plans are based on the National Curriculum and meet the requirements established by the Ministry of Education, Science and Technology.

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The lesson plans will not take the whole term, so use spare time to review material or prepare for exams Teachers can use other textbooks alongside or instead of these lesson plans. Read the lesson plan before you start the lesson. Look ahead to the next lesson, and see if you need to tell pupils to bring materials for next time. Make sure you understand the learning outcomes, and have teaching aids and other preparation ready - each lesson plan shows these using the symbols on the right. Quickly review what you taught last time before starting each lesson. Follow the suggested time allocations for each part of the lesson. If time permits, extend practice with additional work. Lesson plans have a mix of activities for the whole class and for individuals or in pairs. 8 Use the board and other visual aids as you teach. Interact with all pupils in the class - including the guiet ones. Congratulate pupils when they get questions right! Offer 10

Learning outcomes



Preparation

solutions when they don't, and thank them for trying.

Lesson Title: Sorting Objects	Theme: Numbers and Numeration	
Lesson Number: M-09-001	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the lesson,	None	1. Draw on the board (see
pupils will be able to:		example below):
1. Collect and sort objects into		a. 3 large, 3 small stones.
groups.		b. 2 large, 2 small leaves.
2. Describe groups of objects.		c. 2 large, 2 small sticks.
		2. Write the list of objects shown in
		the box in the Independent Practice
		section, on the board.

- 1. **Say**: Please look at the objects on the board. What do you see? Raise your hand. (Answers: stones, leaves, sticks).
- 2. **Say:** Today we will learn about collecting and sorting different objects into groups. We will also describe the groups we have collected.



#### Introduction to the New Material (15 minutes)

- 1. Write this vocabulary list on the top left-hand corner of the board: object, collect, sort, group, property, describe.
- 2. Ask pupils to copy the list into their exercise books.
- 3. **Say**: Sometimes we use words differently in Maths than how we learnt in English. This can often be confusing. But don't worry as you will understand the meaning of these words better as we use them throughout the lesson.
- 4. Point to the objects on the board. **Say**: We want to collect and sort objects according to properties they have in common. We can sort them according to the type of object. We can group all the stones together. We can also group all the leaves together, and all the sticks together.
- 5. If possible, quickly draw all the objects sorted according to type as shown below.
- 6. **Ask**: What common properties have we used to group these objects?
- Give a few moments for pupils to think before asking pupils to raise their hand to give an answer. (Example answers: The objects have been sorted according to the type of object; an object has been sorted according to whether it is a stone, leaf or stick)



Objects collected and sorted into groups according to the type of object

- 8. **Ask**: What property can we use to sort just the stones? Raise your hand. (Example answer: We can sort the stones according to size, large stones together and small stones together).
- 9. Ask: What other properties can we use to collect and sort the objects? Raise your hand.
- 10. Guide the pupils to give other properties. (Example answers: large leaves together and small leaves together; large sticks and small sticks; all large objects and all small objects)

11. Write the following on the board and ask pupils to copy into their exercise books:When we have a <u>collection of objects</u>, we can use a <u>common property</u> to <u>sort</u> them into <u>groups</u>.This property can be used to <u>describe</u> the sort of group we make with the objects.

#### **Guided Practice** (6 minutes)

- 1. Ask pupils raise their hand and call out the names of 8 pupils in the class beginning with the letter A, B or C.
- 2. Write the names on the board: For example, Binta, Aminata, Brima, Charles, Ahmed, Ben, Catherine, Betty.
- 3. Read out the following questions one by one. Ask pupils to work in pairs to discuss and answer the following in their exercise books:
  - a. Look at the collection of names. Sort and list all the names of pupils beginning with B.
  - b. How many pupils have names beginning with B?
  - c. What common property is used to describe the group?
- 4. Walk around, if possible, to check the answers and clear up any misconceptions.
- 5. Have pupils from around the classroom volunteer to give the answers to the questions. Correct any errors. (Answers: a. Binta, Brima, Ben, Betty; b. 4; c. Names beginning with B)

#### Independent Practice (10 minutes)

- 1. Read the list of objects shown in the box on the board.
- 2. **Say**: We can sort the objects on the board into different groups according to a common property.
- 3. **Say**: One group is comprised of fruits. Please raise your hand to give me the name of another group of objects you can see on the list. (Example answers: Colours, furniture,

feelings, kitchen objects, objects for eating. Accept other reasonable answers which describe the objects)

- 4. **Say**: Now work by yourselves to sort the objects on the board into groups.
- 5. Allow pupils 5 minutes to work independently.
- 6. Walk around, if possible, to check the answers and clear up any misconceptions.
- 7. **Say**: Discuss your answers with a partner. Please do not change any answers as we will check them together.
- 8. Allow 1 minute for pupils to discuss and share their ideas.
- 9. Have pupils from around the classroom volunteer to come to the board and write one group of objects. (Example answers: As shown in the table)
   *spoon pink sad bench banana*
- 10. **Ask:** Who has a different idea or answer?
- Discuss other reasonable groupings, for example orange can be put with both banana as fruit, and yellow as colour.
- 12. **Say**: We can put an object in more than one group. It will depend on the property they have in common with other objects.

banana, guava, bench, angry, table, pink, spoon, plum, yellow, sad, blue, knife, chair, happy, cup, orange, plate

spoon pink sad cup yellow ha plate blue an knife orange	y table plum y chair orange guava
--	---

#### Closing (2 minutes)

- 1. **Ask:** What do we do when we want to collect, sort and group objects together? Raise your hand. (Answer: We use a property or properties they have in common.)
- 2. **Say**: Tomorrow we will look at 'sets'. This is the special name given to groups of objects which have been collected and sorted according to a common property.

Lesson Title: Introduction to Sets		Theme: Numbers and Numeration		
Lesson Number: M-09-002		Class/Level	: JSS 3	Time: 35 minutes
Learning Outcomes By the end of the lesson, pupils will be able to identify a set as a well- defined collection of objects or ideas.	None	g Aids	Preparatio 1. Write th the top lef the board: <u>Vocat</u> set, set notation, brackets, list, me an element of.' 2. Write on the b goat, sheep, pend pen. 3. Write the sets the Independent the board	n his vocabulary list on it-hand corner of hulary List well-defined, curly mber, element, 'is oard: Cow, book, cil, paper, pig, ruler, and instructions in Practice section on

- 1. Read these words from the board: Cow, book, goat, sheep, pencil, paper, pig, ruler, pen.
- 2. **Say:** Work in pairs to sort the objects on the board into groups. Use any property they have in common. You have 1 minute.
- 3. After 1 minute, ask pupils to raise their hand and call out the groups of objects and state the common property.

(Example answer: The group including cow, goat, sheep, pig – common property - farm animals; group including book, pencil, paper, ruler, pen – common property - stationery items).

- 4. **Ask:** How many objects are in each group? Raise your hand. (Answer: 4 farm animals; 5 stationery items).
- 5. **Say:** A collection of objects is called a set.
- 6. Write on the board: A collection of objects is called a set.
- 7. **Say:** We will be looking at many aspects of sets over the next 2 weeks. Today's lesson will be an introduction to sets.

#### **Introduction to the New Material** (15 minutes)

- 1. Point to the vocabulary list on the board.
- 2. **Say**: Maths has its own language which can sometimes be confusing. You will understand the meanings of these words better as we use them throughout the lesson.
- 3. **Say:** Please copy the list into your exercise books. You must also take notes during the lesson. We study sets so that we can collect and examine objects and ideas. These objects and ideas share a common property. This helps us to classify and count them.
- Ask pupils to give examples of sets. Guide them to give examples from everyday life. (Example answers: Pupils in JSS3; school subjects; months of the year; days of the week; districts in Sierra Leone)
- 5. **Say:** When we want to talk about sets, we use special language and notation. We will use the set of stationery items to learn some of the basic language and set notation.
- 6. Write on the board: { stationery items }
- 7. Say: We read this as: "the set of stationery items".
- 8. Ask the pupils to read this out loud.

{ stationery items }
read as: "the set of stationery items"

- 9. Say: We use these brackets, { }, called curly brackets to show we are talking about sets.
- 10. Write on the board: S = { stationery items }
- 11. **Say**: We can use a capital letter to represent a set. In our example, we will use S to represent the set of stationery items. We read this as: "S is the set of stationery items." We can also list the objects of the set inside curly brackets.
- 12. Ask: What objects belong to the set of stationery items? Raise your hand to answer.
- 13. Write the answer on the board. (Answer: { book, pencil, paper, ruler, pen })
- 14. **Say:** It does not matter in what order the objects are written. We just need to list all of them. We call the objects in the set members or elements. We separate them using commas.
- 15. Ask: Who can read what the statement says? Raise your hand.
- 16. Guide a pupil who has raised their hand to read the statement out loud. (Answer: "The set whose elements are book, pencil, paper, ruler, pen.")
- 17. Say: We denote the members or elements of the set by this symbol:  $\in$
- 18. Ask: What are the elements of set S? Raise your hand. (Answers: Book, pencil, paper, ruler, pen.)
- 19. Say: We can write that the pen belongs to set S as: pen ∈ S. We read it as: "Pen is an element of set S."

pen ∈ S read as: "pen is an element of set S"

- 20. Say: We can now describe a set properly.
- 21. Write on the board: A set is a <u>well-defined</u> collection of objects or ideas.
- 22. **Say**: If the set is not well-defined, we will not be able to identify all its elements. For example, the set of greatest football players is not well-defined because everybody has their own opinion of what makes a football player great. We cannot say with certainty who should belong to the set.
- 23. Ask: Look again at set S how many elements are in the set? (Answer: 5)
- 24. Write on the board: The number of elements in a set is denoted by n(S).

#### **Guided Practice** (5 minutes)

- 1. Write these 2 sets on the board: A = { vowels }; B = { tall people }
- 2. Guide pupils to say out loud: "A is the set of vowels; B is the set of tall people."
- 3. **Say:** Please work in pairs. Discuss for 1 minute whether or not the sets are well-defined. Remember we should be able to identify all the members of a well-defined set.
- 4. Allow 2 minutes for the pupils to discuss and share their ideas.
- 5. **Say**: Please raise your hand if you think set A is well defined.
- 6. Do a visual check of pupils who have their hands raised.
- 7. Say: Now raise your hands if you think set A is not well defined.
- 8. Take note of pupils who appear unsure and provide assistance during independent practice.
- 9. Most of the pupils should have raised their hands to say set A is well-defined.
- 25. Ask: Why is set A well-defined? Raise your hand.
- 10. Select a pupil from the back of the classroom to answer. (Example answers: We can identify all the elements of the set; we can say or list them with certainty.)
- 11. Have a pupil volunteer to write the set in set notation. (Answer: A = { a, e, i, o, u })
- 12. Most of the pupils should have raised their hands to say set B is **not** well-defined.
- 13. **Ask:** Why is set B not well-defined? (Example answers: We cannot identify all the elements with certainty; we need to specify a minimum height; people may disagree about what tall means, someone who is tall to one person may not be tall to another).

Independent Practice (10 minutes)

- 1. Ask pupils to answer the questions in their exercise books independently.
- 2. Point to these sets and instructions on the board:

Sets: A = { days of the week }; B = { months of the year }

For each set:

- a. Describe the set in words.
- b. Is the set well-defined?

If the set is well-defined:

- c. List the elements of the set;
- d. Select the third element of the set and use set notation to show that it belongs to the set;
- e. Write the number of elements in the set using set notation.
- 3. Walk around the classroom, if possible, and check pupils' work. Give particular attention to the pupils who appeared unsure during the previous activity.
- 4. Ask the pupils to stop after 6 minutes and to discuss their answers with a partner for 1 minute.
- 5. Have pupils from around the classroom volunteer to come to the board and write their answers for the questions. Correct any errors.

(Answers: Set A - a. A is the set of days of the week; b. Yes, A is well defined; c. A = { Monday, Tuesday, Wednesday, Thursday, Friday }; d. Wednesday  $\in$  A; e. n(A) = 7

Set B - a. B is the set of months of the year; b. Yes, B is well defined; c. B = { January, February, March, April, May, June, July, August, September, October, November, December }; d. March  $\in$  B; e. n(B) = 12)

#### Closing (2 minutes)

- 1. Ask: What is a set? Raise your hand.
- Select a pupil from the back of the class to answer. (Answer: A set is a well-defined collection of objects or ideas).
- 3. Make sure pupils have this definition written in their exercise books.
- 4. Ask: How can we tell if a set is well-defined or not? Raise your hand.
- 5. Select a pupil from the middle of the class to answer. (Example answers: We can fully describe the elements of the set; we are able to identify with certainty all the elements of the set.)

Lesson Title: Sets in Real Life	Theme: Numbers and Numeration	
Lesson Number: M-09-003	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None None	Write the lists of items
lesson, pupils will be		from everyday life of a
able to:		JSS 3 pupil from the box in the
1. Identify sets of objects or		Independent Practice section on
ideas from everyday life.		the board.
2. Sort objects or ideas from		
everyday life into sets.		

- 1. **Say:** In the last lesson, we started looking at sets of objects and ideas. Let us see how much we remember.
- 2. Write this set on the board: { colours on the Sierra Leone flag }
- 3. **Say**: List the elements in the set of colours on the Sierra Leone flag using set notation in your exercise books.
- 4. Allow 1 minute for the pupils to answer.
- 5. Have a pupil volunteer to come to the board and write down the answer. (Answer: { green, white, blue })
- 6. **Say:** Remember to always write sets using curly brackets. You may need to practice how to write the brackets until you get used to writing them. Today we are going to investigate sets further by looking at sets from real life.

#### Introduction to the New Material (15 minutes)

- 1. **Say**: Sets can be made from things we can see and touch which are called objects, and things we cannot see and touch which are called ideas. We do not need to worry whether they are objects or ideas as long as we can group them using a common property. We use the general term 'element' or 'member' to identify them.
- 2. Say: Please work in pairs. Discuss and list the elements in the set of 'regions in Sierra Leone'.
- 3. Allow the pupils 1 minute to discuss and write down their answer.
- 4. Have a pupil from the front of the classroom volunteer to write their answer on the board. (Answer: { Eastern Province, Northern Province, Southern Province, Western Area })
- 5. **Say**: It does not matter in what order we write the elements. We just need to list all of them.
- 6. **Ask:** Who remembers what the definition of a set is? Raise your hand.
- 7. Guide the pupil to say: A set is a well-defined collection of objects or ideas.
- 8. **Say**: Now we will write a set about activities people do in their daily lives. One activity which people do on a daily basis is eat. We can call the set D.
- 9. Write on the board: D = { eat, }
- 10. Say: Work with your partner to discuss and add 4 other similar activities from daily life to set D.
- 11. After 2 minutes, have a pupil from the middle of the classroom volunteer to come to the board.
- 12. Say: Add 2 daily activities to set D. (Example answer: D = { eat, sleep, wash })
- Say (to the pupil at the board): Ask the class to give you 2 more activities. Please add them to the set. (Example answer: D = { eat, sleep, wash, cook, clean }).
- 14. Say: Look at the answers on the board.
  - a. Did anyone have different ideas for the activities in D?

- b. Is D well-defined?
- 15. Ask pupils to discuss their ideas in their pairs for 1 minute.
- 16. Ask pupils to raise their hand to share their answers. Discuss all the ideas that are presented. (Pupils may have opposing answers: Daily activities such as play, fetch water, go to the farm, etc., are not on the list so D is not well-defined; D is well-defined, but there are too many elements to list).
- 17. Say: We have a lot of ideas and many of you make good points. D is actually well-defined. However, there are a lot of activities people do on a daily basis, and we cannot list them all. D is a special type of set called an 'infinite set'. We will talk about this type of set next week. For now, we will specify the number of elements we want to list in our sets.

#### **Guided Practice** (5 minutes)

- 1. Ask pupils to continue working in pairs.
- 2. Write on the board:
  - a. Write this list as a set of sauces S: potato leaves, cassava leaves, groundnut soup, crain-crain.
  - b. List 4 elements from the set of soft-drinks, D.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from the left- and right-hand side of the classroom volunteer to write the sets on the board.

(Answer: a. S = { potato leaves, cassava leaves, groundnut soup, crain-crain } Example answer: b. D = { Fanta, Sprite, Coca-Cola, Vimto })

#### **Independent Practice** (10 minutes)

- 1. Read the lists of items from everyday life of a JSS3 pupil from the board (see box below).
- 2. Ask the pupils to sort the items into sets and to identify each set in their exercise books.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Stop pupils after 6-7 minutes to allow them to discuss their answers with a partner.

English	Aunty Memuna	Science
chair	Mathematics	Aunty Fatu
mirror	table	bed
French	lamp	Uncle Saidu
clock	Social Studies	

5. Have pupils raise their hand to share answers.
(Example answers: { relatives } = { Aunty Memuna, Uncle Saidu, Aunty Fatu }; { school subjects } = { French, English, Mathematics, Social Studies, Science }; { bedroom furniture } = { mirror, clock, lamp, table, chair, bed })

#### Closing (2 minutes)

1. **Ask:** What did you learn in this lesson? Raise your hand to answer. (Example answers: About sets in real life; listing elements of a set; sets from real life can have a lot of elements.)

Lesson Title: Describe Sets of Objects	Theme: Numbers and Numeration	
Lesson Number: M-09-004	Class/Level: JSS 3	Time: 35 minutes
		·
Learning Outcomes	Aids Arepa	ration

Learning Outcomes	leaching Alds	Preparation
By the end of the	None None	1. Write this vocabulary
lesson, pupils will be		list on the top left-hand
able to:		corner of the board:
1. Describe sets using words.		Vocabulary List
2. Define the properties of a set		describe, define, list, such that
of objects or ideas.		2. Write the exercise from the
		Independent Practice section on
		the board.

- 1. Write these 2 sets on the board: { beautiful people }, { pupils in JSS3 }
- 2. Ask: Which of these 2 sets is well-defined?
- 3. Allow a few moments for thinking, and then have pupils raise their hand to answer. (Answer: { pupils in JSS3 })
- 4. Ask: Why is the set of beautiful people not well-defined?
- 5. Have 2-3 pupils from around the classroom volunteer to answer. (Answer: We cannot identify with certainty all the elements of the set; someone who may appear beautiful to one person may not to another; we do not know all the beautiful people in the world.)
- 6. **Say**: Today we are going to describe sets using words. We will also define the properties of a set of objects or ideas. We will consider only well-defined sets.

#### Introduction to the New Material (15 minutes)

- 1. **Say**: Write the vocabulary list into your exercise books. We will be using the terms throughout the lesson.
- 2. Give pupils 2 minutes to copy.
- 3. Say: There are several ways we can identify a set of objects.
- 4. **Say**: Let us use the example of the continents of the world. One way of describing the set is by using words. We can say: C is the set of continents of the world.
- 5. Write: C is the set of continents of the world.
- 6. **Say**: Another way to describe a set is by listing the elements of the set.
- Ask: What are the 5 continents of the world? Raise your hand. (Answer: C = { Africa, America, Australia, Asia, Europe })

Describing a Set
C is the set of continents of the world
C = { Africa, America, Australia, Asia, Europe }
C = { continents in the world }

- 8. Ask: Does it matter in what order we list elements? Raise your hand. (Answer: No, as long as we list all of them.)
- 9. Write: C = { Africa, America, Australia, Asia, Europe }
- 10. Say: If there are a lot of elements, we can simply describe the set like this:C = { continents of the world }
- 11. Write: C = { continents of the world }
- 12. Ask the pupils to write the different ways of writing the set C in their exercise books. Give them 2 minutes.

 Say: Another way we can identify a set is by defining the properties of the set. We use a special notation to do this.

- 14. Write:
  - C = { x : x is a continent of the world }
  - C is the set of all x, such that x is a continent of the world
- 15. **Say**: We use a small letter to represent the elements. For this set we have used 'x'. We use the colon symbol : and we read it as "such that".
- 16. Let us look at another example. We will write the set of JSS3 pupils by defining its property.
- 17. Say: Let us call the set J. We will use a small 'p' to represent the elements.
- 18. Guide a pupil to write the set J on the board: J = { p : p is a pupil of JSS3 }
- 19. Ask: How should we read the set written on the board? Answer together.
- 20. Guide pupils to say aloud: "J is the set of all p such that p is a pupil of JSS3."
- 21. Say: Because you are all pupils in JSS3, you are all elements of this set.
- 22. Ask pupils to write the set and how to read it in their exercise books.
- 23. **Say:** This method is very useful when we are describing sets of numbers. We shall do so in tomorrow's lesson. If we are given one of the descriptions of a set, we should be able to write the sets in the other different ways.

#### **Guided Practice** (5 minutes)

- 1. Write on the board: O is the set of oceans of the world. Use *x* for the elements of O.
- 2. Say: Identify O, the set of oceans of the world, using all the ways we have learned today.
- 3. Ask pupils to work in pairs. Allow 3 minutes to answer the question.
- 4. Walk around to check the answers and clear any misconceptions.
- 5. Have pupils from around the classroom volunteer to write their answers on the board. Correct any errors. (Answers: As shown in the box).

O is the set of oceans of the world O = { Arctic, Pacific, Atlantic, Indian, Antarctic } O = { oceans in the world }

 $O = \{x : x \text{ is an ocean of the world} \}$ 

#### Independent Practice (10 minutes)

1. Point to and explain the exercise on the board:

### The following are sets. Use the first letter to represent the set and the second to represent the elements.

- a. Vowels in the English alphabet (V, x)
- b. Colours in a rainbow (C, r)
- c. Letters in the word Maths (L, m)

#### Identify the sets on the board by:

- i. Defining its properties ii. Listing its elements.
- 2. Walk around to check the answers and clear up any misconceptions.
- 3. Ask pupils to exchange exercise books and check each other's work.
- 4. Write the answers on the board. (Answer: a. i. V = { x : x is a vowel }, ii. { a, e, i, o, u };
  b. i. C = { r : r is a colour in a rainbow }, ii. { red, orange, yellow, green, blue, indigo, violet };
  c. i. L = { m : m is a letter in Maths }, ii. { m, a, t, h, s }).

Defining the Properties of a Set C = { x : x is a continent of the world } C is the set of all x, such that x is a continent of the world

#### Closing (3 minutes)

- 1. **Say:** Define the properties of the set of countries in the Mano River Union. Use M for the set and c for the elements of M. Please write your answer on a piece of paper with your name on it. You have 2 minutes.
- Collect the work from pupils at end of the lesson to check their understanding of the topic. (Answer: M = { c : c is a country in the Mano River Union })

Lesson Title: Write Sets of Numbers	Theme: Numbers and Numeration	
Lesson Number: M-09-005	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the	None None	1. Write this vocabulary list on
lesson, pupils will be		the top left-hand corner of the
able to:		board:
1. List the numbers in a set		Vocabulary List
using brackets.		Venn diagram, universal, 'is not an
<ol><li>Identify and interpret set</li></ol>		element of,' complement, less than,
notation.		greater.
		2. Write the exercise from the Guided
		Practice section on the board.
		3. Write the exercise from the
		Independent Practice section on the
		board.

- 1. Write on the board: A = { x : x is a prime number less than 10 }
- 2. Ask the class to read the set out loud together. (Answer: A is the set of all x such that x is a prime number less than 10.)
- 3. **Say**: Today we are going to list the numbers in a set using brackets. We will also identify and interpret set notation.

#### Introduction to the New Material (15 minutes)

- > Write all relevant information on the board as you give its explanation.
- Ask the pupils to copy the information on the board in their exercise books.
- After each question, wait a few moments before selecting pupils from different parts of the classroom to either call out the answer or come to the board and write the answer.
- 1. **Say**: Write the vocabulary list into your exercise books. We will be using the terms throughout the lesson. You have 2 minutes.
- 2. Say: We can identify set A another way using set notation: A = { x : x is prime, x < 10 }</p>
- 3. Write: A = { x : x is prime, x < 10 }
- 4. Ask pupils to copy it in their exercise books.
- 5. **Say**: We read it the same way as before: A is the set of all x such that x is a prime number less than 10.



- 6. **Ask**: What is the property that defines set A? Raise your hand. (Answer: The elements are all prime numbers less than 10.)
- 7. **Say**: List the elements of the set in your exercise books. Remember to use curly brackets.
- 8. After 2 minutes ask pupils to raise their hand to answer. Write the answer on the board. (Answer: A = { 2, 3, 5, 7 }
- 9. **Say**: Write in your exercise books how you would show that 7 belongs to set A.
- 10. After 1 minute ask pupils to raise their hand to answer. Write the answer on the board. (Answer:  $7 \in A$ )
- 11. **Say**: We have one more way to represent a set. We can use a diagram called a 'Venn diagram'.
- 12. Draw the Venn diagram shown on the board and write 'Venn Diagram' under it.
- 13. **Say**: The oval represents the set A. We list all of the elements of set A inside the oval. We sometimes use circles instead of ovals.
- 14. Ask pupils to copy the Venn diagram in their exercise books.
- 15. Say: Please look at the Venn diagram. We have drawn set A inside a rectangle.
- 16. Say: This rectangle is the set from which we have taken the elements of set A.
- 17. Point to the definition of set A.
- 18. **Ask**: Can anyone look at the definition of set A and tell the set that we have taken its elements from?
- 19. Guide the pupils to the part which shows that the elements have been taken from the set of numbers less than 10. (Answer: A = { x : x is prime, x < 10 })
- 20. **Say**: This set is called the 'universal set'. We define it as the set of all the elements under consideration. We represent it with the letter U. We will assume that the set has no fractions or decimals less than 10, just whole numbers.
- 21. Write on the board: U = { x : x < 10 }
- 22. Ask the pupils to read this out loud. (Answer: U is the set of all x such that x is less than 10.)
- 23. Write 'U' on the Venn diagram next to the rectangle. See the diagram below.
- 24. **Say:** Please copy the definition of set U in your exercise books. List all its elements in set notation.
- 25. Allow time for pupils to copy and write down the answers.
- 26. Ask pupils to raise their hand to answer. Write the answer on the board. (Answer:  $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ )
- 27. Ask: What numbers can you see in set U which are not in set A?
- 28. Ask pupils to think for a minutes and then raise their hand to answer. (Answers: 0, 1, 4, 6, 8, 9.)
- 29. **Say**: We use this symbol ∉ to show that a number does not belong to set A.
- 30. Write on the board:  $6 \notin A$ .
- 31. Say: This is read as "6 is not an element of A".
- Say: We refer to the set of elements in set U which are not in set A as the <u>complement of A</u>. We denote this by the symbol A<sup>c</sup>.
- 33. Say: Note the spelling of the word 'complement'.
- 34. Write on the board: A<sup>c</sup> = { 0, 1, 4, 6, 8, 9 }
- 35. **Say**: We can show A<sup>c</sup> on the Venn diagram.
- 36. **Write** the numbers in A<sup>c</sup> in the Venn diagram (as show in the box). Ask the pupils to copy this in their exercise books.





37. Say: It is not necessary to put A<sup>c</sup> in its own oval as this can become confusing when we have more than one set on the same Venn diagram.

#### **Guided Practice** (5 minutes)

- 1. Point to and read the exercise on the board:
  - $U = \{x : x \text{ is a counting number, } x < 10 \}$
  - a. Define B, the set of all even numbers y less than 10.
  - b. List all the elements in set B using set notation.
  - c. Write using set notation:
    - i) an element in set B.
    - ii) an element in set B<sup>c</sup>.
- 2. Ask pupils to work in pairs to discuss and write the answers.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to share answers. (Answer: a. B = { y : y is even, y < 10 }; b. B = { 2, 4, 6, 8 }; c. i) Any one of these:  $2 \in B, 4 \in B, 6$  $\in$  B, 8  $\in$  B); ii) Any one of these: 1  $\in$  B<sup>c</sup>, 3  $\in$  B<sup>c</sup>, 5  $\in$  B<sup>c</sup>, 7  $\in$  B<sup>c</sup>, 9  $\in$  B<sup>c</sup>)

#### **Independent Practice** (10 minutes)

- 1. Point to and read out the exercise on the board:
  - $U = \{ 1, 4, 9, 16, 25, 36, 49, 64, 81 \}$
  - a. Describe U, the universal set in words.
  - b. Define the set of square numbers less than 30. Represent the set by A, the elements by s.
  - c. List the elements of set A.
  - d. Draw the Venn diagram for the set.
  - e. Write the elements of A<sup>c</sup> in the Venn diagram.
- 2. Ask the pupils to answer independently in their exercise books.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to write the answers on the board. Correct any errors.
  - (Answer: a. U = { square numbers less than 100 }; b. A =
  - $\{s:s is a square number, s < 30\};$
  - c. A = { 1, 4, 9, 16, 25 }; d. see box; e. see box)

#### **Closing** (3 minutes)

16 81 49

 $A = \{ s : s \text{ is a square number, } s < 30 \}$ 

9

4 1

25

36

U

Α

64

- 1. Ask: What is the universal set, U? Raise your hand. (Answer: The universal set, U is the set of all elements under consideration.
- 2. Ask: What is the complement of a set, A<sup>c</sup>? Raise your hand. (Answer: The complement of a set, A<sup>c</sup> is the set of elements in set U which are not in set A).

Lesson Title: Finite Sets	Theme: Number and Numeration	
Lesson Number: M-09-006	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
lesson, pupils, will be	None	board:
able to identify sets of objects,		{ Sunday, Monday, Tuesday, Wednesday,
things, ideas and numbers that		Thursday, Friday }
are finite.		2. Write this vocabulary list on the top
		left-hand corner of the board:
		Vocabulary List: finite, ellipsis
		3. Write the exercise from the Guided
		Practice section on the board.
		4. Write the exercise from the
		Independent Practice section on the
		board.

- 1. **Say**: Please raise your hands if you recognise the set on the board. (Answer: The set of the days of the week.)
- 2. Say: Today we are going to identify sets of objects, things, ideas and numbers that are finite.

#### Introduction to the New Material (10 minutes)

- 1. Say: Let us call the set on the board, D.
- 2. Ask: How many elements are in set D? Raise your hand. (Answer: 7)
- 3. **Say:** There are 7 days in a week. We will always be able to list the days and count how many there are. We call a set where we can list and count all the elements or members a 'finite' set.
- 4. **Write** the definition of a finite set on the board: A set where we can list and count all the elements or members.
- 5. Ask the pupils to copy the information on the board in their exercise books.
- 6. Have a pupil from the left-hand side of the classroom volunteer to remind the class how to write the number of elements in set D using set notation. (Answer: n(D) = 7)
- 7. Write on the board: L = { letters in the English alphabet }
- 8. Ask: How can we tell if this is a finite set?
- 9. Give a few moments for the pupils to discuss this with a partner.
- 10. Have a pupil from the front of the classroom volunteer to answer. (Answer: We can list and count all its elements.)
- 11. Ask: How many elements are in set L? Raise your hand. (Answer: n(L) = 26)
- 12. **Say:** Because we have so many elements in set L, we may not want to list all of them. We can use 'ellipsis' in the middle of the list to show that some elements are missing.
- 13. Write on the board:  $L = \{a, b, c, d, ..., x, y, z\}$
- 14. Say: Please list enough elements so the ones missed out are clearly understood.

#### **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Point to and read the exercise on the board:
  - a. A = { factors of 10 }

- b. B = { whole numbers less than 100 }
- c. C = { multiples of 3 between 30 and 50 }
  - i. State whether the set is finite or not.
  - If the set is finite:
  - ii. List its elements. Use ellipsis if the set has more than 6 elements.
  - iii. Write the number of elements in the set using set notation.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give the various answers to the problem.
- 5. Correct any errors.

Answers:	a.i. finite	ii. A = { 1, 2, 5, 10 }	iii.	n(A) = 4
	b.i. finite	ii. B = { 0, 1, 2,, 97, 98, 99}	iii.	n(B) = 100
	c.i. finite	ii. C = { 33, 36, 39, 42, 45, 48 }	iii.	n(C) = 6

#### **Independent Practice** (10 minutes)

1. Point to and read the following exercise from the board for pupils to complete independently.

```
U = { whole numbers from 1 to 50 }
```

List the elements in the following sets and give the number of elements in each set. Use ellipsis if there are more than 8 elements in the set.

- a. A = { prime numbers }
- b. B = { square numbers }
- c. C = { numbers divisible by 5 }
- d. D = { multiples of 4 }
- 2. Give pupils 7 minutes to complete the exercise.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from the front, back, left-hand side and right-hand side of the classroom volunteer to give the answers. Write the correct answers on the board. Ask pupils to check their work

Answers:	a. A = { 2, 3, 5, 7,, 37, 41, 43, 47 }	n(A) = 15
	b. B = { 1, 4, 9, 16, 25,36, 49 }	n(B) = 7
	c. C = { 5, 10, 15, 20,, 35, 40, 45, 50 }	n(C) = 10
	d. D = { 4, 8, 12, 16,, 36, 40, 44, 48 }	n(D) = 12

#### **Closing** (3 minutes)

- 1. **Ask**: What is a finite set? Raise your hand. (*Answer*: A set where we can list and count all the elements or members.)
- 2. Say: Great work! In the next lesson, we will look at infinite sets.

Lesson Title: Infinite Sets	Theme: Number and Numeration	
Lesson Number: M-09-007	Class/Level: JSS 3 Time: 35 minu	

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None None	1. Write this vocabulary
lesson, pupils will be		list on the top left-hand
able to identify sets of objects,		corner of the board:
things, ideas and numbers that		Vocabulary List: infinite
are infinite.		2. Write the exercise from the
		Independent Practice section on
		the board.

- 1. Write on the board: { factors of 2 }
- 2. Say: Please list the element of the set of factors of 2.
- 3. Allow a few moments for the pupils to list the elements of the set.
- 4. Ask: How many elements do we have in the set? Raise your hand. (Answer: 2 elements, { 1, 2 })
- 5. **Say**: Raise your hand if you know the name of this type of set.
- 6. Select a pupil who has raised their hand to answer. (Answer: finite set.)
- 7. Ask: What is a finite set?
- 8. Have a pupil at the front of the classroom volunteer to answer. (Answer: A finite set is a set where we can list and count all the elements or members.)
- 9. Say: Today we are going to identify sets of objects, things ideas and numbers that are infinite.

#### Introduction to the New Material (13 minutes)

- 1. Write on the board: { multiples of 2 }
- 2. Say: Give me examples of members of this set.
- 3. Allow pupils from around the classroom to give examples of multiples of 2.
- 4. Write them on the board as they call them out. Allow this to continue until you have 8-10 examples of multiples of 2. (Example answers: 2, 4, 6, 8, 10, 12, 14, 16)
- 5. **Ask:** How many elements do we have so far?
- 6. Have a pupil volunteer to count the examples on the board and give the answer.
- 7. Ask: What other elements can you think of? (Example answers: 18, 20, 22)
- 8. **Say:** The set of multiples of 2 is an example of an infinite set.
- 9. Ask: From what we have done so far, who can explain what an infinite set is?
- 10. Have 2-3 pupils volunteer to give explanations. (Example answers: An infinite set is a set where there are many elements; we cannot list all the elements; there are too many elements to list)
- 11. **Write** on the board: An infinite set is a set where we <u>cannot</u> list or count all the elements or members. No matter how many elements we list, there will always be more.
- 12. Enclose the set of numbers on the board with curly brackets and add ellipsis at the end: { 2, 4, 6, 8, 10. 12, ... })
- 13. **Say:** We add ellipses at the end of infinite sets to show that the elements go on forever. This is different from when we added ellipsis to finite sets. In a finite set, we know the number of elements. We add ellipsis to show some elements have been left out. In an infinite set, we do not know the number of elements. We add ellipsis to show that there are infinitely more elements to come.
- 14. Ask pupils to copy the information on the board in their exercise books.

#### **Guided Practice** (5 minutes)

- 1. Write on the board: A = { 1, 2, 3, 4, ... 18, 19, 20 }, B = { 1, 2, 3, 4, ... }
- 2. Say: Please write down which set you think is a finite set and which one is an infinite set.
- 3. Allow pupils 1 minute to answer.
- 4. **Say**: Turn to your neighbour and discuss your answers with them. You have 1 minute.
- 5. Say: Raise your hand if you think set A is a finite set.
- 6. Do a visual check of hands raised and take note of pupils who seem unsure.
- 7. Say: Raise your hand if you think set A is an infinite set.
- 8. Do a visual check of hands raised and again take note of pupils who seem unsure.
- 9. **Ask**: Do you think set B is finite or infinite?
- 10. Allow pupils to call out answers.
- 11. Have 2-3 pupils volunteer to give a reason for their answers for sets A and B.
- 12. **Say**: A is a finite set because the elements go from 1 to 20. The ellipsis shows we have not listed some elements. B is an infinite set because the elements start from 1 and continue for ever. The ellipsis shows that we can continue writing elements until infinity.
- 13. Write this on the board and ask pupils to copy this in their exercise books.

#### **Independent Practice** (10 minutes)

1. Point to and read the following exercise:

Sets:			
a.	{ even numbers from 20 to 30 }.	b.	{ prime numbers}
с.	{ odd numbers between 10 and	d.	( whole numbers greater than
100	) }	20	00 }

For each set:

- i. Say whether the set is finite or infinite.
- ii. List the elements of the set using set notation.
- 2. Ask the pupils to answer independently in their exercise books.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions, particularly with the pupils who seemed unsure during the previous exercise.
- Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: a. i. finite, ii. { 20, 22, 24, 26, 28, 30 }; b. i. infinite, ii. { 2, 3, 5, 7, ... }; c. i. finite, ii. { 11, 13, 15, 17, ... 95, 97, 99 }; d. i. infinite ii. { 2001, 2002, 2003, 2004, ... }).

#### Closing (2 minutes)

1. **Ask:** What did you learn in this lesson? (Example answers: About infinite sets; the difference between finite and infinite sets; the difference between using ellipsis for finite and infinite sets)

Lesson Title: Unit and Empty Sets	Theme: Number and Numeration	
Lesson Number: M-09-008	Class/Level: JSS 3 Time: 35 minut	

Learning Outcomes	<b>A</b> Teaching Aids	A Preparation
By the end of the	None	1. Write this vocabulary
lesson, pupils will be		list on the top left-hand
able to identify a unit set as one		corner of the board:
with one element, and an		<u>Vocabulary List:</u> null
empty set as one with no		2. Write the questions from the
elements.		Independent Practice section on
		the board.

- 1. Write on the board: C = { even prime numbers }, D = { even prime numbers greater than 5 }
- 2. Say: List the element of sets C and D.
- 3. Allow 1 minute for the pupils to list the elements of the sets.
- 4. Ask: How many elements do we have in each set? Raise your hand to answer. (Answers: C = { 2 }, n(C) = 1; D = { }, n(D) = 0)
- 5. **Say**: Today we are going to be looking at unit and empty sets.

#### Introduction to the New Material (12 minutes)

1. Write:

 $C = \{2\}, n(C) = 1 - Unit set (has only 1 element)$ 

D = { }, n(D) = 0 – Empty or null set (has no elements). Denoted by : { } or  $\emptyset$ 

- Say: There are special names given to sets C and D. Because there is only one element in set C, it is called a 'unit set'. Because set D has no elements, it is called an 'empty set' or a 'null set'. It is written as { } or Ø.
- 3. Ask pupils to copy the information in their exercise books.
- 4. **Say:** An example of a set with only one element from everyday life is a set of months with 28 days.
- 5. Ask: What month has 28 days?
- 6. Have a pupil volunteer to answer. (Answer: February)
- 7. Say: Let us call the set of months with 28 days E.
- 8. Write on the board: E = { February }, n(E) = 1.
- 9. Say: An example of an empty set from everyday life is a set of months with 32 days.
- 10. Write on the board: { months with 32 days } =  $\emptyset$ , n( $\emptyset$ ) = 0.
- 11. **Say**: Remember the set written as { } is an empty set. It contains nothing. The set written as {  $\emptyset$  } is not an empty set. It is a unit set containing one element which is the empty set.
- 12. Discuss this for 1 minute. Make sure the pupils understand the difference between { } and {  $\emptyset$  }.

#### **Guided Practice** (5 minutes)

- 1. Write on the board:
  - a. Is an empty set a finite or infinite set?
  - b. Is a unit set a finite or infinite set?
- 2. Ask pupils to work in pairs to answer the questions.
- 3. Walk around, if possible, to check answers and clear any misconceptions about what makes a set finite or infinite.

4. Have one pupil from the left-hand side and one from the right-hand side of the classroom volunteer to give their answers to the questions. (Answers: a. finite, it has 0 elements; b. finite, it has 1 element.)

#### **Independent Practice** (10 minutes)

- 1. Read the questions below from the board.
  - i. Give one example from everyday life of: a) an empty set b) a unit set
  - ii. Give one example from Maths of: a) an empty set b) a unit set
- 2. Ask the pupils to work independently to answer the questions in their exercise books.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Ask the opinion of the class after each answer and discuss the answer for a few moments. (Example answers:
  - i. a) { dogs with 5 legs }; { year with 13 months }
    - b) { months with 29 days }, { February }; { days starting with the letter M }, { Monday }
  - ii. a) { whole numbers which are both even and odd }; { squares with unequal sides }
    - b) { whole numbers which have only one factor}, { 1 }; { whole numbers with 4 as their square }, { 2 })
- 6. Clear up any misconceptions. For example, the fact that 2 is not the same as { 2 }. Explain that 2 is the number 2, while { 2 } is the set which has the number 2 as its only element.

#### Closing (5 minutes)

- 1. Say: Please say 2 things you learned during this lesson.
- 2. Have 2-3 pupils volunteer to say 2 things they learned. (Example answers: I learned about unit and empty sets; I learned that both unit and empty sets are finite; February can have both 28 and 29 days)
- 3. Discuss and ask follow-up questions. Example follow-up questions: What did you learn about unit sets? What did you learn about empty sets? How many elements are in unit and empty sets? Is there any other month where the number of days changes like February?

Lesson Title: Equal Sets	Theme: Number and Numeration	
Lesson Number: M-09-009	Class/Level: JSS 3 Time: 35 minu	

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None None	1. Write on the board: {
lesson, pupils will be		4, 2, 1, 3 }, { 3, 2, 1, 4 }, {
able to identify sets with the		2, 1, 2, 3, 4 }
same elements.		2. Write the questions from the
		Independent Practice section on
		the board.

- 1. Point to the sets on the board: { 4, 2, 1, 3 }, { 3, 2, 1, 4 }, { 2, 1, 2, 3, 4 }
- 2. Ask: Look at the sets on the board. What do you notice?
- 3. Give pupils 1 minute to discuss the sets with their partner.
- 4. Guide pupils to say that the elements of all 3 sets are 1, 2, 3, 4.
- 5. Say: In today's lesson, we will be looking at equal sets.

#### Introduction to the New Material (15 minutes)

- 1. Say and write on the board: 2 sets are equal when they have exactly the same elements.
- 2. Say: We can also compare more than 2 sets.
- 3. Write on the board: { 4, 2, 1, 3 } = { 3, 2, 1, 4 } = { 2 , 1, 2, 3, 4 }.
- 4. **Say** and **write** on the board: It does not matter what order we write the elements or if we write the same element more than once. When counting the number of elements in a set, we only count the unique elements. This means we do not count any element more than once.
- 5. Ask: How many unique elements do the 3 sets have exactly? Raise your hand. (Answer: All 3 sets have exactly 4 unique elements 1, 2, 3, 4.
- 6. **Say**: We count the '2' in the third set only once. It is also good practice to put the elements in ascending or descending order. We can now write one set to represent all 3 sets on the board.
- 7. Write on the board: All 3 sets can be represented by { 1, 2, 3, 4 }.
- 8. Ask pupils to write down all the information on the board in their exercise books.
- 9. Write on the board: A = { 2, 4, 6, 8 }, B = { 10, 20, 30, 40, 50 }.
- 10. Say: Look at the sets on the board. What can you say about the elements in the sets?
- 11. Give pupils 2-3 minutes to discuss and share ideas with their partner.
- 12. **Say:** Describe the elements in sets A and B. Raise your hand to answer. (Example answers: The elements in set A are even numbers less than 10 (or multiples of 2 less than 10), the elements in set B are multiples of 10 from 10 to 50.)
- 13. **Ask**: Do the sets have exactly the same elements? Raise your hand to answer. (Answer: No, the elements in the sets are all different.)
- 14. Say: 2 sets are not equal when they have different elements.
- 15. Write on the board:  $\{2, 4, 6, 8\} \neq \{10, 20, 30, 40, 50\}$
- 16. Say and write : This can also be written as A  $\neq$  B.
- 17. Ask the pupils to copy all the new information on the board in their exercise books.

#### **Guided Practice** (5 minutes)

1. Write on the board: C = { 1, 3, 5, 7 }, D = { 3, 5 }

- 2. Say: Discuss with your partner if you think sets C and D are equal.
- 3. Allow 1-2 minutes for the discussion.
- Select 2-3 pupils who have their hands raised to answer. (Answers: Set C is not equal to set D; they do not have the same number of unique elements; n(C) = 4, n(D) = 2; the 2 sets do not have exactly the same elements)
- 5. Clear up any misconceptions.
- 6. **Say**: The most important thing for sets to be equal is that they must have 'exactly' the same elements.

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the following questions on the board:
  - i. Put = or  $\neq$  between the 2 sets to make the statement true.
    - a. { prime numbers less than 10 } [ { 2, 3, 5, 7 }
    - b. {1,3,5} [ {1,2,3}
    - c. { a, a, b } 🗌 { a, b }
  - ii. Are A and B equal?
    - a. A = { first 4 positive whole numbers }; B = { 4, 2, 1, 3 }
    - b. A = { s : s is a square number, s < 20 }; B = { 1, 4, 9, 16, 25 }
- 2. Walk around, if possible, to check the answers and clear up any misconceptions.
- 3. Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: i. a. =; b.  $\neq$ ; c. =; ii. a. Yes; b. No).

#### Closing (3 minutes)

- 1. Ask: What is the most important thing for sets to be equal?
- 2. Ask pupils to write their names and answers on a piece of paper and to hand them in. (Answer: Equal sets must have exactly the same elements.)
- 3. Use this to determine whether pupils achieved the main learning outcome of the lesson. Review at start of the next lesson.

Lesson Title: Equivalent Sets	Theme: Numbers and Numeration	
Lesson Number: M-09-010	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None	1. Write this vocabulary list on the
lesson, pupils will be		top left-hand corner of the board:
able to:		Vocabulary List: equivalent, unique
1. Identify that an equivalent		2. Write the questions from the
set has the same number of		Independent Practice section on the board.
elements.		3. Write on the board: { a, b, c }, { b, a, b, c,
2. Distinguish between equal		a, c, b }
and equivalent sets.		

- 1. Point to the sets on the board: { a, b, c }, { b, a, b, c, a, c, b }
- 2. **Ask**: Are these 2 sets equal? Raise your hand.
- 3. Select a pupil from the back of the classroom to answer. (Answer: Yes).
- 4. **Ask:** What makes them equal? Raise your hand. (Example answers: They have exactly the same elements. The elements for both sets are a, b, c).
- 5. Say: Today we will be looking at equivalent sets and learning how they differ from equal sets.

#### Introduction to the New Material (15 minutes)

- 1. Write on the board: A = { a, b, c }, B = { 1, 2, 3 }.
- 2. Ask: What do you notice about the 2 sets on the board?
- 3. Guide the pupils to say the 2 sets have the same number of elements.
- 4. Write on the board: n(A) = n(B).
- 5. **Say** and **write** on the board: When 2 sets have the same number of elements they are called 'equivalent' sets. We write this using a double-headed arrow as: A ↔ B.
- 6. Say: We read it as set A is equivalent to set B.
- 7. Write: C = { 5, 10, 15 }, D = { 5, 15, 5, 10 }.
- 8. **Say**: Please look at sets C and D. Can you say if they are equal sets, equivalent sets, neither or both?
- 9. Ask pupils to think about this for 2 minutes and write down any ideas they have.
- 10. Guide them to think about the number and type of unique elements in each set.
- 11. Ask pupils to discuss their ideas with their partner for 2 more minutes.
- 12. Have pupils from around the classroom volunteer to share their ideas with the class. Discuss these ideas. (Example answers: Set C has 3 unique elements 5, 10 and 15. Set D also has 3 unique elements and can be written as  $\{5, 10, 15\}$ . Therefore the 2 sets are equal because they have exactly the same elements. Sets C and D are equivalent because they have the same number of elements n(C) = n(D) = 3).
- 13. Write on the board: All equal sets are equivalent but not all equivalent sets are equal.
- 14. Discuss the above statement using open-ended questions for 2 minutes using sets A, B, C and D.
  - a. Which sets are equivalent? Why?
  - b. Which sets are equal? Why?

(Answers: Sets A, B, C and D all have 3 unique elements so they are all equivalent with each other. Sets A and B have exactly the same 3 unique elements, so they are equal).

#### **Guided Practice** (5 minutes)

- 1. Write on the board: A = { w, x, y, z }, B = { cow, goat, sheep, pig }, C = { w, x, y, x. w, y, z }.
- 2. Ask the pupils to discuss in pairs which of these sets are equal and which are equivalent to each other.
- Have pupils from around the classroom volunteer to give their answers.
   (Answers: A ↔ B ↔ C because they all have 4 unique elements, n(A) = n(B) = n(C) = 4; A = C because they have exactly the same 4 unique elements, w, x, y, z).

#### **Independent Practice** (10 minutes)

- 1. Point to the following questions on the board for pupils to answer independently:
  - The 4 sets M, N, O and P are described as shown:
  - M = { kite, rectangle, rhombus, square, trapezium }, N = { odd numbers between 1 and 10 },
  - O = { square numbers between 1 and 30 }, P = { 1, 3, 5, 7, 9 },
    - i. List the elements for the sets N and O.
    - Use ↔ (is equivalent to) or = (is equal to) between the 2 sets to make the statements true.
    - a. M 🗌 N b. O 🗌 P c. N 🗌 P d. M 🗌 P
- 2. Have pupils from around the classroom volunteer to give their answers to the questions.
- 3. Discuss the answers and clear any misconceptions (in particular ii. c.).

#### Closing (3 minutes)

- 1. Ask: What is the difference equal and equivalent sets?
- 2. Have a pupil from the back of the classroom volunteer to answer.
- Have another pupil from the front of the classroom volunteer to repeat the answer given. (Answer: Equal sets have exactly the same elements while equivalent sets have the same number of elements.)

Lesson Title: Introduction to Subsets	Theme: Numbers and Numeration	
Lesson Number: M-09-011	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	A Teaching Aids	<b>Preparation</b>
By the end of the	None	1. Write this vocabulary list on the
lesson, pupils will be		top left-hand corner of the board:
able to identify subsets as a		<u>Vocabulary List -</u> subset
collection of objects within a		2. Write on the board: X = { whole numbers
set.		between 1 and 10 }, Y = { even numbers
		between 1 and 10 }
		3. Write the exercise from the Independent
		Practice section on the board.

- Say: List the elements of the following sets on the board: X = { whole numbers between 1 and 10 }, Y = { even numbers between 1 and 10 }.
- 2. Allow time for the pupils to list the elements of the 2 sets.
- 3. Ask pupils to raise their hand to share answers. **Write** the answers on the board. (Answer: X = { 2, 3, 4, 5, 6, 7, 8, 9 }, Y = { 2, 4, 6, 8 }
- 4. Ask: Which elements in set Y are present in set X? Raise your hand. (Answer: { 2, 4, 6, 8 })
- 5. Say: Today we will introduce subsets where all the elements in one set are present in another.

#### Introduction to the New Material (15 minutes)

- 1. **Say:** If 'every' element in set Y is present in set X, then Y is a subset of X. We use a special notation for this relationship between the 2 set.
- 2. Write on the board:

$$\left\{ \begin{array}{l} 2,\,4,\,6,\,8 \end{array} \right\} \subset \left\{ \begin{array}{l} 2,\,3,\,4,\,5,\,6,\,7,\,8,\,9 \end{array} \right\} \\ Y \, \subset \, X \\ \end{array}$$

This is read as set Y "is a subset of" set X

- 3. Say: If 'every' element in a set, e.g. W, is not present in set X, then W is not a subset of X.
- 4. Write on the board.

W = { 5, 6, 7, 8, 9, 10 }

 $W \not\subset X$ 

- 5. Ask: There is one element in set W which is not present in set X. What element is that?
- 6. Allow pupils to call out the answer. (Answer: 10).
- Say: For set W to be a subset of set X, 'every' element in W must be present in X. We can draw a Venn diagram showing the relationship between sets X and Y.
- 8. Draw the Venn diagram as shown in the box.
- Say: Note how all the even numbers are shown in both X and Y. If we remove the oval shape for set Y, all the elements in X will still be shown. The set X has many subsets, not just Y. One other example is { 2, 3, 9 }.
- 10. Say: Write down 3 more subsets of set X in your exercise books.
- 11. After 2 minutes ask pupils to raise their hand to answer. **Write** some example answers on the board. (Example answers: { 3, 5, 7, 9 }, { 8, 9 }, { 7 }, { 2 }, { 2 , 3 }, { })



- 12. Write on the board: { }  $\subset$  X.
- 13. Ask: How can you explain the statement, "the empty set is a subset of set X?"
- 14. Discuss this statement with the pupils. (Discussion point: The empty set has nothing in it, and one of the subsets of set X also has nothing in it. This set with nothing in it is an empty set. Therefore the empty set is a subset of X.)
- 15. **Say** and **write** on the board: The empty set is a subset of every set, because there will always be one subset which has nothing in it.
- 16. Ask pupils to copy the above statement in their exercise books.

#### **Guided Practice** (5 minutes)

- 1. Write on the board: S = { 6, 12, 18, 24, 30 }, T = { 6, 9, 12 }
- 2. Ask: Look at the sets on the board is set T a subset of set S?
- 3. Allow the pupils some time (less than a minute) to think about the question.
- 4. Say: Please discuss your answer with your neighbour.
- 5. Allow 1 minute for the pupils to discuss their answer with their neighbour.
- 6. Have a pupil from the front of the classroom volunteer to give their answer.
- 7. Have 1-2 other pupils from around the classroom volunteer to say if they agree/disagree with the answer and why.
- 8. Allow the class to discuss the answer for 1 minute and come to an agreement about the answer.
- 9. Share the answer with the class. (Answer: T ⊄ S, because the element 9, which is in set T, is not in set S).

#### **Independent Practice** (10 minutes)

- 1. Point to the following exercise on the board:
  - i.  $A = \{ 21, 22, 23, 24, 25 \},$  ii.  $A = \{ 10, 20, 30, 40, 50 \},$  iii.  $A = \{ 1, 3, 6, 10, 15 \}$  $B = \{ 31, 32, 33, 34, 35 \}$   $B = \{ 30, 50 \}$   $B = \{ 21, 28, 36, 45, 55 \}$ 
    - a. Use set notation to write whether B is a subset of A.
    - b. Draw the Venn diagram if B is a subset of A.

iv. Write all the subsets you can find for the set { a, b, c }.

- 2. Ask the pupils to answer the questions independently in 6 minutes.
- 3. Walk around the classroom to check on their answers and clear up any misconceptions.

(Answers: i. a. B ⊄ A; ii. a. B ⊂ A iii. a. B ⊄ A only 1 Venn diagram required for ii. b. shown right. iv. { }, { a }, { b }, { c }, { a, b } { a, c }, { b, c }, { a, b, c }.

4. Discuss the answers to Question iv, especially why { } and { a, b, c } are subsets of { a, b, c }. (Discussion point: Empty set { } is discussed above in the introduction. In the case of { a, b, c }, one of the subsets of any set, A, is the set itself because every element of set A is an element of A).

#### Closing (2 minutes)

1. **Ask:** What is the condition for set P to be a subset of set Q? Raise your hand. (Answer: If 'every' element in set P is present in set Q, then P is a subset of Q).



Lesson Title: Identifying Subsets of a Set of Real Numbers		Theme: Numbers and Numeration		
Lesson Number: M-09-012			Class/Level: JSS 3	Time: 35 minutes
Learning Outcomes By the end of the lesson, pupils will be able to identify subsets of real numbers: natural numbers, whole numbers, rational numbers (integers, fractions and decimals).	Teaching Aids None	Voca ratio 2. W -2, 3. W a. Dr relat	<b>Preparation</b> 1. Write this vocabula left-hand corner of the <u>bulary List:</u> natural num- nal numbers, irrational rite this set of numbers 0, $\frac{3}{4}$ , 0.5, $\sqrt{3}$ , $\pi$ }. rite these instructions of aw each subset according ionship found in the pre-	ary list on the top ne board: nbers, integers, numbers. on the board: { 5, on the board: ng to the evious exercise.
		b. La c. Lis	bel each subset with th t a few examples of ele	e correct symbol. ments which
		belor	ng in each subset.	

- 1. Read this set of numbers from the board: { 5, -2, 0,  $\frac{3}{4}$ , 0.5,  $\sqrt{3}$ ,  $\pi$  }. (Leave the list on the board to use throughout the lesson).
- 2. Say: Each number on the board has got a name that we identify it by. For instance,  $\frac{3}{4}$  is called a fraction.
- 3. Ask: What name do we give the number 0.5? Raise your hand. (Answer: Decimal).
- 4. Say: In today's lesson, we will learn how to identify subsets of real numbers.

#### Introduction to the New Material (15 minutes)

- 1. **Say:** All the numbers we will look at today are subsets of real numbers. The set of real numbers is the universal set for the sets we are considering. It is denoted by R.
- 2. Write: R is the universal set for all real numbers.
  - R = { real numbers }
- 3. **Say**: All the subsets of R are infinite sets so we will only list a few elements for each. We will use ellipsis to show how infinite sets go on forever.
- 4. Ask: How is an ellipsis written? Raise your hand. (Answer; as 3 dots . . .)
- 5. **Say**: The first subset is the set of natural numbers. These are the counting numbers we learnt in primary school.
- 6. Ask: What are the first 5 elements of this set? Raise your hand. (Answer: 1, 2, 3, 4, 5).
- 7. If a pupil gives 0 as an example, **ask**: Why is 0 not a counting number? (Example answers: We always start counting objects from 1; we never start counting from 0)
- 8. Write: N = { natural numbers }
  - = { 1, 2, 3, 4, 5, . . . }
- 9. Say: The next subset is the set of whole numbers, W. These are the natural numbers plus 0.
- 10. Write: W = { whole numbers }

= { 0, 1, 2, 3, 4, 5, . . . }

- 11. Point to the number -2 from the list at the start of the lesson.
- 12. Ask: What is this type of number called? Raise your hand. (Answer: negative integer).

- 13. **Say**: The set of integers is made up of positive natural numbers, 0, and negative natural numbers. It is denoted by Z.
- 14. Write: Z = { integers }

= { . . ., -3, -2, -1, 0, 1, 2, 3, . . . }

- 15. Explain to pupils how the ellipsis at both ends of the set shows the numbers carry on forever in both positive and negative directions.
- 16. **Ask**: What numbers can you see on the board that we have not discussed as yet? Raise your hand. (Answers:  $\frac{3}{4}$ ; 0.5;  $\sqrt{3}$ ;  $\pi$ ).
- 17. Say: We already know that  $\frac{3}{4}$  is a fraction and 0.25 is a decimal number. Both these numbers belong to the set of rational numbers, Q.
- 18. Write: Q = { rational numbers }

 $= \{ \ldots, -2, -11/2, -0.5, 0, 1, 33/4, \ldots \}$ 

All integers, fractions and decimals are rational numbers. However, he denominator cannot be 0.

- 19. **Say**: 'Rational' comes from the word 'ratio' and describes how all the numbers can be written as ratios or fractions of integers.
- 20. Write:  $\frac{3}{4}$  is a ratio of the integers 3 and 4.
- 21. Write: 0.5 can be written as a fraction  $\frac{5}{10}$  which is the same as  $\frac{1}{2}$ .
- 22. Ask: What 2 integers make up this fraction? (Answer: 1 and 2).
- 23. Explain that the denominator of rational numbers cannot be 0 as we cannot divide by 0.
- 24. **Ask**: What about the integers themselves? Can they be rational numbers as well? Raise your hand.
- 25. Guide the pupils to see how an integer, e.g. 5, can be written as the fraction  $\frac{5}{1}$ .
- 26. **Say**: All integers, fractions and decimals are rational numbers. However, the denominator cannot be 0. We have just listed a few in the example.
- 27. **Say:** The last 2 numbers on the list are  $\sqrt{3}$  and  $\pi$ . They belong to a set of numbers which cannot be written as fractions.
- 28. Explain to pupils that  $\pi$  written as  $\frac{22}{7}$  or 3.142 is just an approximate value. In reality, its value goes on forever. It is 3.141592653... to 9 decimal places.
- 29. Write: 3.141592653... to 9 decimal places.
- 30. Point out the ellipsis.
- 31. **Say**: Numbers whose values go on forever like that are called irrational numbers. You will learn more about irrational numbers in senior secondary school.

#### **Guided Practice** (5 minutes)

- 1. **Say**: Look at all the sets on the board. We can show in set notation the relationship between each of them. All the elements in the set of natural numbers N also belong to the set of whole numbers W.
- 2. Ask: How would we write this? Raise your hand. (Answer:  $N \subset W$ ).
- 3. Say: Work in pairs to write the relationships between the other subsets of R.
- 4. Walk around the class to check on answers and clear any misconceptions.
- 5. Have a pupil from the left-hand side of the classroom volunteer to come to the board to write the relationship between the sets W and Z. (Answer:  $W \subset Z$ ).
- 6. Ask: Who can write the complete relationship between R and all its subsets? Raise your hand.
- 7. Select a pupil with their hand raised to do so on the board.
- 8. Ask the pupil at the board to explain how they arrived at their answer.

9. Ask the other pupils if they agree. Ask them to make comments and any corrections to what is on the board.

(Answer:  $N \subset W \subset Z \subset Q \subset R$ ; the sets all form subsets as shown; all the elements in N belong to W; all the elements in W belong to Z; all the elements in Z belong to Q; all the elements in Q belong to R.

10. Write: N  $\subset$  W  $\subset$  Z  $\subset$  Q  $\subset$  R

Natural numbers are whole numbers, whole numbers are integers and integers are rational numbers. They are all part of the real number system which we use everyday.

11. **Say**: Natural numbers are whole numbers, whole numbers are integers and integers are rational numbers. These all form the real number system which we use in Mathematics and in everyday life.

#### **Independent Practice** (10 minutes)

- 1. **Say**: A good way to remember these subsets of R is by representing them all on the same Venn diagram.
- 2. Say: Draw R, the universal set as a rectangle.
- 3. Show this on the board.
- 4. Read these instructions from the board and ask the pupils to complete them independently in their exercise books:
  - a. Draw each subset according to the relationship found in the previous exercise.
  - b. Label each subset with the correct symbol.
  - c. List a few examples of elements which belong in each subset.
- 5. Give pupils 7 minutes to complete the activity.
- 6. Walk around to check answers and clear misconceptions.
- 7. Have pupils from around the classroom volunteer to give their answers to the questions.
- 8. Write the correct answer on the board. Ask pupils to correct their work.

(Answer: See box - shown with a few elements listed as examples. If the pupil puts an irrational number in any of

the subsets, show how we can represent it within R for now as shown).

#### Closing (3 minutes)

- 1. **Say:** List all the subsets of R to which '4' and '-4' belong. You can use the definitions or the Venn diagram to help.
- 2. Ask pupils to raise their hand to answer. (Answer: 4 belongs to the sets of natural numbers, whole numbers, integers and rational numbers. –4 belongs to the sets of integers and rational numbers).



Lesson Title: Comparing Sets of Real Numbers	Theme: Number and Numeration	on
Lesson Number: M-09-013	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None	1. Write this vocabulary list
lesson, pupils will be		on the top left-hand corner
able to:		of the board:
1. Compare sets of real		Vocabulary List: data, intersection,
numbers.		union
2. Use a Venn diagram to		2. Write the exercise from the
compare sets of real numbers.		Guided Practice section on the
		board.
		3. Write the exercise from the
		Independent Practice section on
		the board.

- 1. **Say:** In the last lesson, we looked at the subsets of the set of real numbers. Who can name one of the subsets? Raise your hand.
- 2. Select pupils with their hands raised to give the answers. (Answers: Counting or natural numbers, whole numbers, integers, rational numbers, irrational numbers (optional)).
- 3. Say: Give me an example of a rational number.
- 4. Have a pupil from the back of the classroom volunteer to answer. (Answer: 0.3, 7,  $-\frac{1}{r}$ )
- 5. Discuss incorrect answers.
- 6. **Say**: Today we are going to compare sets of real numbers. This will include how to use Venn diagrams to compare sets of real numbers.

#### Introduction to the New Material (15 minutes)

- 1. **Say**: We compare sets of numbers to determine the similarities and differences between them. This is useful in fields such as science, banking, and accounting where large amounts of data are processed. Data are 'collections' of objects and ideas that have been sorted into groups.
- 2. Ask: What does the definition of data remind you of? Raise your hand.
- 3. Give the pupils some time to think and then select a pupil who has raised their hand. (Answer: It is similar to that of a well-defined set. So, data are well-defined sets).
- 4. Say: One way to compare sets of data is to examine the lists of elements from the sets.
- 5. Write on the board: A= { 1, 3, 5, 7, 9 }, B = { 1, 2, 3, 4, 5 }
- 6. **Say**: There are special symbols and vocabulary we use in set notation to show the similarities and differences between sets.
- 7. Ask: Which elements are in set A that are also in set B? Raise your hand.
- 8. Give the pupils some time to think and then select a pupil who has raised their hand. (Answer: 1, 3, 5).
- 9. Write on the board:

{ elements in both sets A and B } = { 1, 3, 5 } A  $\cap$  B = { 1, 3, 5 } read a

read as "A intersection B" or the "intersection of sets A and B"
10. Ask: Which elements are in either set A or set B? Raise your hand. (Answer: 1, 2, 3, 4, 5, 7, 9)

# 11. Write on the board:

 $\{ elements in either set A or set B \}^{=} \{ 1, 2, 3, 4, 5, 7, 9 \}$ 

 $A \cup B = \{ 1, 2, 3, 4, 5, 7, 9 \}$  read as "A union B"

or the "union of sets A and B"

- 12. **Say**: Look at the union of sets A and B. Discuss with your partner what you think the universal set might be. (Example answer: U = { 1, 2, 3, 4, 5, 6, 7, 8, 9 }.
- 13. Say: Discuss with your partner what the elements of the complement of set A are.
- 14. Have 2-3 pupils from around the classroom volunteer to give their answers.
- 15. Ask the class if they agree. You may get 2 answers to this question:

 $A^{c} = \{2, 4\}$  and  $A^{c} = \{2, 4, 6, 8\}$ . Discuss the difference between the 2 answers. (Discussion point:  $A^{c} = \{2, 4\}$  is the set of numbers which are in B but not in A. The complement of a set means 'all' the elements not in the set – we need to consider the elements in the universal set as well. Correct answer:  $A^{c} = \{2, 4, 6, 8\}$ ).

- 16. **Ask**: What is the complement of set B? Raise your hand. (Answer: B<sup>c</sup> = { 6, 7, 8, 9 }).
- 17. Say: We can show all this information on a Venn diagram.
- 18. Draw the Venn diagram shown.
- 19. **Say**: It is now clear from the Venn diagram what the elements of sets A<sup>c</sup> and B<sup>c</sup> are.

# **Guided Practice** (5 minutes)

- 1. Point to the following exercise on the board:
  - U = { 1, 2, 3, 4, 5, 6 }, C = { 1, 2, 3, 4 }, D = { 4, 5 }. a. Draw a Venn diagram of the given information.
    - b. Write in set notation:
    - $i. \ C \cap D \quad ii. \ C \cup D \quad iii. \ C^C \qquad iv. \qquad D^C$
- 2. Ask pupils to work in pairs to discuss and share ideas. Give them 3 minutes.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: a. see Venn diagram in box;
- 5. Ask pupils to correct their work as needed.

b. ii.  $C \cap D = \{4\}$  iii.  $C \cup D = \{1, 2, 3, 4, 5\}$  iv.  $C^{C} = \{5, 6\}$  v.  $D^{C} = \{1, 2, 3, 6\}$ Independent Practice (10 minutes)

- 1. Point to the following exercise on the board:
  - U = { natural numbers less than 10 } R = { squares less than 10 }

S = { even numbers less than 10 }

- T = { odd squares less than 10 }
- a. List the elements of the given sets.
- b. Draw the sets on a Venn diagram.
- $c. \quad \mathsf{Find} \ R \cap S \qquad \quad \mathsf{d}. \ \ \mathsf{Find} \ R \cup S$





- e. Is  $R \subset T$ ? f. Is  $T \subset R$ ?
- 2. Ask the pupils to answer independently in their exercise books. Give them 7 minutes.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.

(Answers: a.  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$   $R = \{1, 4, 9\}$   $S = \{2, 4, 6, 8\}$   $T = \{1, 9\}$ 



c.  $R \cap S = \{4\}$  d.  $R \cup S = \{1, 2, 4, 6, 8, 9\}$  e. No,  $R \not\subset T$  f. Yes,  $T \subset R$ 

- 1. Ask pupils the questions below. Allow them some time to think and then have a pupil volunteer to answer.
- Ask: What does the intersection of the 2 sets A and B show? (Answer: { elements in both sets A and B })
- Ask: What does the union of 2 sets show? (Answer: { elements in either set A or set B })

Lesson Title: Ordering Sets of Real Numbers			Theme: Number and Numeration			
Lesson Number: M-09-014			Class/Level: JSS 3	Time: 35 minutes		
	Learning Outcomes	Teaching	Aids	<b>Prepa</b>	ration	
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Learning Outcomes	I eaching Alds	A
By the end of the	None None	Rone None
lesson, pupils will be		
able to order sets of real		
numbers.		

- 1. Ask: Can someone tell the class what we did in the last lesson?
- 2. Select a pupil who has their hand raised to answer. (Example answers: Compared sets of numbers; found intersection and union of sets; drew Venn diagrams)
- 3. Ask: What is the intersection of 2 sets?
- 4. Have a pupil from the back of the classroom volunteer to answer. (Answer: It is the set of elements common to both sets.)
- 5. Ask: What is the union of 2 sets?
- 6. Have a pupil from the front of the classroom volunteer to answer. (Answer: It is the set which has all the elements from both sets.)
- 7. Say: In the last lesson, we only compared sets from natural numbers. We can compare other subsets of the real numbers in a similar way. Today we are going to order sets of real numbers.

# **Introduction to the New Material** (10 minutes)

- 1. Write on the board: { 4, -2, 0, 9, -4 }
- 2. Ask: What set do these numbers belong to? Raise your hand. (Answer: Integers).
- 3. Say: Please put the set of integers in ascending order. Raise your hand. (Answer: {-4, -2, 0, 4, 9 **})**.
- 4. Remind pupils that negative integers get smaller the further away from 0 they are. Positive integers get larger the further away from 0 they are. Draw a quick number line to show this (see example).



- 5. Write on the board:  $\langle , \rangle, \leq , \geq$
- 6. Say: We can use these symbols to compare 2 numbers. You know them from previous years.
- 7. Ask pupils to raise their hand to identify the symbols. (Answer: less than, greater than, less than or equal to, greater than or equal to).
- 8. Write on the board:  $\frac{3}{5} \square \frac{4}{7}$ .
- 9. Say: Put one of these symbols between these 2 numbers to make the statement true.
- 10. Have a pupil volunteer to put the right symbol on the board and explain how they got their answer. (Answer: >; possible explanation - use LCM to make a

common denominator and order according to numerator,  $\frac{3}{r} = \frac{21}{2r}, \frac{4}{7} =$  $\frac{20}{35}, \frac{21}{35} > \frac{20}{35}$ 

- 11. Write on the board: { 0.3, 1.35, 0.25, 1.257 }.
- 12. Ask pupils to work in pairs to put this set of decimal numbers in ascending order.
- 13. Allow them 2 minutes to do this work.
- 14. Ask: Who can explain to the class what to do?

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0	2	5	
0	3		
1	2	5	7
1	3	5	

- 15. Select a pupil who has raised their hand to answer. (Example answer: Look at the individual digits of each number one at a time and compare them with each other. Smaller digits are less than larger digits.)
- 16. Use a place value chart to help explain the process.
- 17. Ask the pairs to check their answers and make any corrections. (Answer: { 0.25, 0.3, 1.257, 1.35}).
- 18. Say: We can also order a mixture of real numbers.
- 19. Write on the board:  $\{1.5, \frac{2}{5}, 6, 0.35\}$
- 20. **Say**: Put this set of numbers in descending order. Raise your hand. (Answer: { 6, 1.5,  $\frac{2}{5}$ , 0.35 }. Change the fraction  $\frac{2}{5}$  to a decimal 0.4).

# **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs to put the sets of real numbers in ascending order.
- 2. Write on the board:

a. { 3, -5, 1, -3 } b.	$\{\frac{3}{5}, \frac{2}{5}, \frac{1}{2}, \frac{4}{7}\}$	с.	{ 3.4, -3.4, 3.04, 0.34 }
------------------------	--	----	---------------------------

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Ask pupils to correct any errors. (Answers: a. {-5, -3, 1, 3}; b.  $\{\frac{2}{5}, \frac{4}{7}, \frac{1}{2}, \frac{3}{5}\}$ ; c. {-3.4, 0.34, 3.04, 3.4}.

## Independent Practice (10 minutes)

- 1. Ask the pupils to answer independently.
- 2. Write on the board:

Put the following sets of numbers in ascending order:

a. 
$$\{-2, \frac{2}{5}, 3, 2.5\}$$
 b.  $\{0.35, -0.25, \frac{1}{2}, -0.5\}$ 

Put the following sets of numbers in descending order:

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Ask pupils to correct any errors. (Answers: a.  $\{-2, \frac{2}{5}, 2.5, 3\}$ ; b.  $\{-0.5, -0.25, 0.35, \frac{1}{2}\}$ ; c.  $\{2.4, 2.04, 0.24, -2.4\}$ ; d.  $\{4, 2, 0, -3, -6\}$ ).

- 1. Say: Please tell the class one new thing you learned today.
- 2. Have pupils from around the class volunteer to answer. (Example answers: How to order integers (or fractions or decimals or mixture of real numbers.)

Lesson Title: Real Numbers on a Number Line	Theme: Numbers and Numeration			
Lesson Number: M-09-015	Class/Level: JSS 3	Time: 35 minutes		
Learning Outcomes	Aids Prepa	ration		
By the end of the Mone	( 🕺 1. Wri	te on the board:		

lesson, pupils will be able to locate real numbers on a	None	$= \{-6.7, 6, -0.67, -\frac{6}{7}\}$
number line.		used throughout lesson.
		2. Draw a number line from -10
		to +10 (see example below).

- 1. Say: Please put the elements of the set in ascending order.
- 2. Have a pupil from the front of the classroom volunteer to explain their solution to the class. (Answer: { -6.7,  $-\frac{6}{7}$ , -0.67, 6 }).
- 3. Ask: What do you notice about the negative numbers? Raise your hand.
- 4. Have a pupil from the back of the classroom volunteer to answer. (Answer: Negative numbers are smaller the further they are from 0).
- 5. Say: Today we are going to locate real numbers on a number line.

# Introduction to the New Material (10 minutes)

- 1. **Say**: All real numbers can be located on a number line. We can draw a number line to show only integers, only decimals or fractions or a mixture of numbers.
- 2. Write on the board: { -6, 3, 6, -3 }
- 3. Say: Can someone show the class where these numbers fit on the number line?
- 4. Have a pupil volunteer to show these with an **X** on the number line on the board.



- 6. **Ask**: What do you notice about the positions of the 3 and -3 in relation to 0? Raise your hand. (Answer: They are the same distance away from 0).
- 5. **Say**: Every real number has positive and negative counterparts. They are the same distance away from zero. They will always equal to 0 when we add them together.
- 6. Write:

3 + (-3) = 0 three plus negative three equals zero  $\frac{1}{2} + \left(-\frac{1}{2}\right) = 0$  half plus negative half equals zero

- 7. **Say**: Also notice that numbers get smaller and smaller as we move left on the number line. They get larger as we move towards the right.
- 8. Show this on the number line as above.
- 9. Say: Let us now look at how to put decimal numbers on a number line.
- 10. Write on the board: Show 2.37 on a number line.
- 11. Say: It is not easy to put this on the number line on the board.

- 12. Show how difficult it is to accurately put this number on the number line because we have to guess where it fits between 2 and 3.
- 13. Say: We need a number line with a different scale to show this.
- 14. Draw the number line below. Ask a pupil to show where to mark 2.37 on it. (Answer: As shown.)



- 15. **Say:** It is not always practical to have different scales, especially if we want to fit different types of numbers on the same number line. Most times we can only try to be as accurate as possible. Let us try to locate our first set of numbers on the number line. { -6.7, 6, -0.67,  $-\frac{6}{7}$  }
- 16. Draw a number line as shown. Select pupils to mark the numbers on the number line.



#### **Guided Practice** (10 minutes)

- 1. Say: Please work in pairs to answer the questions on the board.
- 2. Write on the board:

Draw number lines marked from -7 to +7. Locate the set of real numbers on the number line

a. 
$$\{3, -0.25, -5, 4.5\}$$
  
b.  $\{\frac{3}{8}, 0.375, \frac{15}{7}, -6.2, -6\frac{2}{7}\}$ 

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to show their answers on the number lines.
- 5. **Ask:** How did you decide where the fractions fitted on the number line. Raise your hand. (Example answers: By changing them to decimal numbers; changing them to mixed numbers; finding common denominators.)
- 6. Ask pupils to correct any errors. (Answers: As shown below)



#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Write on the board: Mark the set of real numbers on a number line

$$\left\{ 4, -6\frac{7}{10}, -1\frac{11}{15}, 3\frac{3}{4}, -6.7, -3, 0, 2.5, -\frac{67}{10} \right\}$$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions. (Answer: As shown below)
- 6. Ask pupils to correct any errors.



- 1. **Say:** Please listen to this statement. Raise your left hand if you think it is true, raise your right hand if you think it is false. The number to the right on the number line is always greater than the one on the left.
- 2. Allow a few moments for pupils to think, and raise their hands.
- 3. Have a pupil to volunteer give their reason for their choice. Ask the class if they agree.
- 4. Show on a number line that a number on the right of another is bigger, e.g. 6 > 4, -3 > -5.

LESSOI	The Roman Numeral System	meme. Number and Numeration				
Lesson Number: M-09-016			Class/Level: JSS 3			Time: 35 minutes
	Learning Outcomes	N AA	Teaching Aids	A	Prepar	ation
	By the end of the lesson, pupils will		None		1. lf po	ossible, bring an
	be able to:				object	such as a clock

Themes Number and Numeration

**Opening** (2 minutes)

Roman numeral.

- 1. Point to the Roman numerals on the board. **Say**: Please raise your hand if you have seen numbers written using these symbols on the board before.
- 2. If no one raises their hands, **ask:** What do the symbols look like? Raise your hand. (Example answers: Roman numerals; symbols for counting.)
- 3. **Say**: Today we are going to identify the symbols used to show a Roman numeral. You will be able to read, write and count numbers up to 20. Romans are people who lived in Rome in the country of Italy.

# Introduction to the New Material (15 minutes)

Lesson Title, The Doman Numeral System

1. Identify the symbols used to show a

2. Read, write and count numbers up to 20.

- 1. **Say**: The Romans wrote their numbers in a different way from what we are used to. Their numbers are used on clocks, watches and similar objects. Roman numerals were invented by the ancient Romans between the years 900 and 800 B.C.
- 2. Show the object with Roman numerals to the class, if available.
- 3. **Say**: The Romans used 7 basic symbols to represent their numbers. We can match these numbers to our base 10 numbers. Our base 10 numbers are derived from the Hindu-Arabic numbers.
- 4. Write the information in the box on the board.
- 5. **Say**: These symbols can be written in capital or small letters. They mean the same thing.
- 6. Write on the board: i, ii, iii
- 7. **Ask**: Where have you seen these numbers before in the classroom? (Answer: For questions on the board).
- 8. **Say:** We can write or work out other numbers from these 7 Roman numerals.
- 9. **Say**: Look at the numbers on the table. Work in pairs and see if you can write the first 5 numbers of the Roman number system.
- 10. Allow pupils 1 minute to answer the question.
- 11. Have a pupil from the right-hand side of the classroom volunteer to come to the board to write their answer.
- 12. The pupil may write 4 = IIII, or 4 = IV. Ask if anyone has a different idea.
- 13. Allow pupils to share their ideas.
- 14. **Say**: The Romans never wrote more than 3 of the same symbols together. They always tried to find a different way to show the number instead.
- 15. Write on the board: 4 = 5 1 IV
- 16. **Say**: The 'I' in front of 'V' means subtraction. Whenever a smaller numeral is in front of a larger numeral we subtract.

Symbols used in Roman Numerals					
Hindu- Arabic Roman					
1	I				
5	V				
10	Х				
50	L				
100	С				
500	D				
1000	М				

which shows Roman numerals.

2. Write on the board:

( I, II, III, . . . }

- 17. Have pupils raise their hand to share the first 5 numbers. (Answers: 1 = I, 2 = II, 3 = III, 4 = IV, V = 5).
- 18. Say: The Romans made numbers by adding or subtracting like this.
- 19. Write the box on the right, on the board.
- 20. **Say:** Work with your partner. Follow the same method to calculate the numbers 8 to 11. Remember, we cannot have more than 3 of the same symbols together.
- 21. Allow the pupils 3 minutes to discuss and share their ideas on writing the numbers.
- 22. Have pupils from around the classroom volunteer to come to the board to complete the answers.
- 23. Discuss the answers given. Point out the 'I' in front of the 'X,' which means subtract. (Answers: 8 = 5+1+1+1 = VIII; 9 = 10-1 = IX; 10 = X; 11 = 10+1 = XI).

#### **Guided Practice** (5 minutes)

- 1. **Say**: Continue to work in your pairs. Show how you would calculate the Roman numerals for the numbers 12-14.
- 2. Walk around, if possible, to check answers and clear misconceptions.
- 3. Have pupils from around the classroom volunteer to come to the board to complete the answers.
- 4. Discuss the answers given and ask pupils to correct any errors in their work.
  (Answers: 12 = 10+1+1 = XII; 13 = 10+1+1+1 = XIII; 14 = 10 + 5 1 = XIV).

#### **Independent Practice** (10 minutes)

- 1. Write the table on the right on the board.
- 2. As you write, ask the pupils to work independently to copy and complete the table.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers.
- 5. Write the correct answers and ask pupils to correct any errors in their work.

(Answers: 5 = V; 10 = X; 11 = 10+1 = XI; 12 = 10+2 = XII;

$$17 = 10+7 = XVII; 18 = 10+8 = XVIII; 19 = 10+9 = XIX).$$

- 1. **Ask:** How did the Romans work out how to write the number 19?
- 2. Have a pupil from the back of the classroom volunteer to answer. (Answer: 19 = 10 + (10 1) = 10 + 9 = XIX).
- 3. Say: Tomorrow, we will look at how to write larger Roman numerals. Great job, class!

4	=	5 - 1	=	IV
5	=	5	=	V
6	=	5 + 1	=	VI
7	=	5 + 1 + 1	=	VII

r						
Numerals from 1-20						
Hindu-Arabic	Calculation	Roman				
1	1	1				
2	1 + 1	11				
3	1 + 1 + 1					
4	5 – 1	IV				
5						
6	5 + 1	VI				
7	5 + 1 + 1 = 5 + 2	VII				
8	5 + 3	VIII				
9	10 - 1	IX				
10						
11						
12						
13						
14	10 + (5 - 1) = 10 + 4	XIV				
15						
16						
17						
18						
19						
20	10 + 10	XX				

Lesson Title: Converting between Base 10 and	Theme: Numbers and Numeration			
Roman Numerals				
Lesson Number: M-09-017	Class/Level: JSS 3 Time: 35 minutes			
Learning Outcomes / Teaching /	lids A Prena	ration		

	Learning Outcomes	ьĦл	leaching Alds	AL	Preparation
	By the end of the lesson,		None		Write the number 22 on
	pupils will be able to				the board.
convert numbers up to 100					
from base 10 to Roman					
numerals and vice versa.					

- 1. Ask the pupils to convert the number on the board to Roman numerals.
- 2. **Say**: Please raise your hand as soon as you finish.
- After 1 minute, have a pupil volunteer to show their answer and how they got it, on the board. (Example answers: 22 = 10 + 10 + I + I = XXII; 22 = 10 + 10 + 2 = XXII; 22 = 20 + 2 = XXII)
- 4. **Say**: Today we are going to convert numbers up to 100 from base 10 to Roman numerals, and from Roman numerals to base 10.

# Introduction to the New Material (15 minutes)

- 1. Say: In the Hindu-Arabic system, which numerals are used to make the numbers from 1 to 100?
- 2. After a few moments, allow pupils to call out the answers. (Answers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
- 3. Ask: Which symbols are used to make the Roman numerals from 1 to 100?
- After a few moments, allow pupils to call out the answer. (Answer: 1 = I; 5 = V; 10 = X; 50 = L, 100 = C)
- 5. **Say:** All the numbers from 1 to 100 have to be made from just these 5 numerals.
- 6. Write on the board: 10, 20, 30, 40, 50, 60, 70, 80, 90.
- 7. Say: Work in pairs to write the Roman numbers for the Tens numbers.
- 8. Allow 3 minutes for the pupils to calculate the Tens numbers.
- 9. Stop pupils after 3 minutes. Discuss their answers by asking open-ended questions. For example: a. How did you calculate the number 40?
  - b. Does anyone have a different idea or method?
  - c. Did you have difficulties with calculating 60? What were they?
  - d. Explain how the calculation for the number 90 is similar to that for the number 9.

(Example answers: a. 40 = 50 - 10 = XL; b. Allow pupils to share any different ideas or methods. Check and clear any misconceptions, (e.g. that 40 is equal to XXXX);

c. Discuss any difficulties pupils may have, clear up any misconceptions.

60 = 50 + 10 = LX;

d. 9 = 10 - 1 and 90 = 100 - 10. If we multiply the calculation for 9 with 10 we get the calculation for 90. The answer is XC. Ask pupils to see if this works for all the other Tens numbers. (Yes, it does.) Explain to pupils that this is clear from how numbers are written in base 10, i.e. 9 and 90. This shows that 90 is 10 times bigger than 9. This is not so clear in the way numbers are written in the Roman system, i.e. IX and XC)).

10. **Write** the answers for 10, 20, 30, 40 and 50 on the board. (Answers; 10 = X; 20 = XX; 30 = XXX; 40 = XL; 50 = L).

11. Have pupils from around the classroom volunteer to write the answers for 60, 70, 80 and 90 on the board.

(Answers: 60 = LX; 70 = LXX; 80 = LXXX; 90 = XC).

- 12. Say: Now call out any number from 20 to 40. We will convert it to a Roman numeral together.
  - a. Allow pupils to call out numbers.
  - b. Write a few on the board.
  - c. Select one to work through together on the board. Use the table from the last lesson.
  - d. Discuss any problems.
    (Example answer: 28 = 20 + 8 = XX + VIII = XXVIII; another possibility is: X+X+V+I+I+I)
- 13. Write on the board: Convert this number to base 10: XLI
- 14. Allow 1 minute for pupils to carry out their calculation.
- 15. Have one pupil from the front volunteer to come to the board and show their calculation.
- 16. Ask if there are any different ideas or methods. Check and clear up any misconceptions. (Example answers: XLI = 40 + 1 = 41; XLI = 50 10 + 1 = 41; XLI = (50 10) + 1 = 41).
- 17. **Say**: Please remember to use brackets as it will help to make the calculation clear.

# **Guided Practice** (5 minutes)

- 1. Write on the board:
  - a. Convert 44 to Roman numerals. b. Convert XCII to Hindu-Arabic (base 10) numbers. Show your calculations.
- 2. Ask the pupils to work in pairs to convert the numbers.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have one pupil from the left- and one from the right-hand side of the classroom volunteer to give their answers.
- 5. Write the correct answers on the board. (Answers: a. 44 = 40 + 4 = XLIV; accept other calculations,

e.g. 44 = (50 - 10) + (5 - 1); b. XCII = 90 + 2 = 92; accept other calculations, e.g. (100-10) + 2 = 92)

#### **Independent Practice** (10 minutes)

- 1. Write the following numbers on the board:
  - a. Convert these base 10 numbers to Roman numerals: 49, 74, 99
  - b. Convert these Roman numerals to base 10: XXXIV, LIV, LXXXVIII. Show your calculations.
- 2. Ask the pupils to work independently to answer the questions.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to show their calculations on the board.
- 5. Accept all accurate calculations. Correct any errors. (Example answers: 49 = 40+9 = XLIX; 74 = 70+4 = LXXIV; 99 = 90-9 = XCIX; XXXIV = 30+4 = 34; LIV = 50+4 = 54; LXXXVIII = 80+8 = 88)

- 1. Ask pupils to write their name on a piece of paper.
- 2. Ask them to write the conversions for the numbers 48 and XCIV on the piece of paper.
- 3. Collect the paper from each pupil at the end of the lesson to check their understanding.

Lesson	Title: Introduction to Base	2		Theme: Nu	mber ar	nd Numeratio	on
Lesson	<b>Number</b> : M-09-018			Class/Leve	l: JSS 3		Time: 35 minutes
	Learning Outcomes	N AA	Teaching A	lids	AI	Preparation	
$(((\bigcirc)))$	Dutha and of the lasson	WIЛ	Nono			1 Drow the	dat nattarn halaw

By the end of the lesson,	None	1. Draw the dot pattern below,
pupils will be able to:		
1. Identify the numerals used to		
read and write in base 2.		
2. Count up to 20 in base 2 and		2. Write the vocabulary list on the top
work out the pattern of		right-hand corner of the board -
numbers.		Vocabulary List: binary, device

- 1. Write these numbers on the board: { 1, 2, 4, 8, 16, ... }
- 2. **Say**: Please look at the pattern on the board. Discuss the pattern with your seatmate. Raise your hand if you can tell the next number in the pattern.
- 3. Allow a few moments for discussion and for pupils to raise their hands.
- 4. If no hands are raised, **say:** The numbers form a pattern based on doubling. Look at the first 3 numbers. Use it to help you find the next number.
- 5. Have a pupil from the back of the classroom volunteer to answer. (Answer: 32)
- 6. **Say**: Today we are going to learn how to identify the numerals used to read and write in base 2. We will also learn how to count up to 20 in base 2 and work out the pattern of numbers.

# Introduction to the New Material (15 minutes)

- 1. Ask: Which numerals do we use when we count in base 10?
- 2. Allow pupils to call out the answers. (Answers: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)
- 3. Say: There are 10 numerals in base 10. These are 0 to 9.
- 4. Ask: How many numerals do you think will be in base 2? Raise your hand. (Answer: 2)
- 5. Ask: Which numerals do you think we will use to count in base 2?
- 6. Guide pupils to say 0 and 1.
- 7. **Say**: Base 2 numbers are also called 'binary' numbers. They are called binary numbers because 2 different digits are used. All base 2 numbers are made using the 2 numerals, 0 and 1.
- 8. **Say:** We use base 2 or binary numbers in all our mobile phones, laptops, computers, DVDs, etc. All the information in these devices is stored and sent in 0s and 1s.
- 9. **Say**: When we learned how to count in base 10, we used Units, Tens, Hundreds, Thousands and so on. These are all powers of 10.
- 10. **Say**: To count with base 2, we will use the powers of 2. Work in pairs. Show that these numbers,  $\{1, 2, 4, 8, 16, ...\}$  are powers of 2
- 11. Allow pupils time to work out the powers of 2. Have pupils from around the classroom volunteer to call out the answers. (Answers:  $1 = 2^0$ ;  $2 = 2^1$ ;  $4 = 2^2$ ;  $8 = 2^3$ ;  $16 = 2^4$ ).
- 12. Say: To help us understand how base 2 numbers work, we are going to make the set of numbers on the board. We need 5 pieces of paper – tear them out from your exercise books. Mark the dots on your paper as shown on the board.

16	8	4	2	1		
					=	1
					=	2
					=	3
					=	4
					=	5

- 13. **Say**: Let us make the first 5 numbers using our dot paper. We will use base 10 numbers to help us keep track.
- 14. Say: Draw a table as shown on the board. Arrange your dot papers in the order shown.
- 15. **Say**: When we use a number, we will turn the dot paper face up. We will write 1 underneath the number.
- 16. **Say**: When we are not using a number, we will turn the dot paper face down. We will write 0 underneath the number.
- 17. Say: Let us start with 1. Which dot papers should be face up?
- 18. Guide pupils to see that to make 1, they need to turn up the dot paper with one dot. All the others should be turned over. See the guide at the end of the lesson plan for the first 5 numbers.
- 19. Ask pupils to fill in the first row of their table as shown below.
- 20. **Say**: In base 2, we can read this as "no 16s, no 8s, no 4s, no 2s, one 1. So, 1 = 1. Let us now use our dot papers to make 2.
- 21. Allow the pupils to discuss and share ideas with their partner.
- 22. Have a pupil volunteer to come to the board and show how they made 2 using their dot papers. This is shown in row 2 of the table.
- 23. Say: In base 2 we do not read this number as ten. Instead we say "one zero".
- 24. Write on the board:

#### 2 = 10 one zero

- 25. Repeat these steps for the numbers 3 to 5.
- 26. **Write** on the board how to say each number shown by the pupil:
  - 3 = 11 one one
  - 4 = 100 one zero zero
  - 5 = 101 one zero one

16	8	4	2	1		
0	0	0	0	1	=	1
0	0	0	1	0	=	2
0	0	0	1	1	=	3
0	0	1	0	0	=	4
0	0	1	0	1	=	5

27. Say: Note how you are adding the powers of 2 to make the numbers from 1 to 5.

#### **Guided Practice** (5 minutes)

- 1. Ask the pupils to work in pairs.
- 2. Ask them to use their dot papers to make the numbers 6 to 10 in base 2.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions. They can ignore the extra zeros in front of the numbers.

16	8	4	2	1		
		1	1	0	=	6
		1	1	1	=	7
	1	0	0	0	=	8
	1	0	0	1	=	9
	1	0	1	0	=	10
	•	•	•	•	•	•

- 5. **Say**: Please look at the numbers on the table. Can you see a pattern in the numbers?
- 6. Guide pupils to say that the numbers on the table go up in the pattern:
  1 number has 1 digit; 2 numbers have 2 digits, 4 numbers have 3 digits.
  This is the same as the original set on the board {
  1, 2, 4, ... }.
- 7. **Ask**: How many numbers will you expect to have 4 digits if we continue the table? (Answer: 8)
- 8. **Say**: We already have 3. Let us see how many more we will find.

16	8	4	2	1		
	1	0	1	1	=	11
	1	1	0	0	=	12
	1	1	0	1	=	13
	1	1	1	0	=	14
	1	1	1	1	=	15
1	0	0	0	0	=	16
1	0	0	0	1	=	17
1	0	0	1	0	=	18
1	0	0	1	1	=	19
1	0	1	0	0	=	20

# **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to complete the table from 11 to 20.
- 2. Walk around, if possible, to check the answers and clear up any misconceptions.
- 3. Have pupils from around the classroom volunteer to give their answers to the questions. (Answers: as shown in the table.)
- 4. Ask: Did we get the numbers we expected to have 5 digits?
- 5. Allow the pupils some time to count the numbers with 5 digits and then raise their hand to answer. (Answer: Yes, there are 8 numbers with 5 digits.)

- 1. **Say:** Please raise your hand if you know the pattern in making base 2 numbers.
- 2. Have a pupil who has raised their hand to answer the question.
- 3. Guide the pupil to explain that there is only one number with one digit, 2 numbers with 2 digits, 4 with 3, 8 with 4 and so on. This follows the same pattern as the numbers we get with the set of powers of 2 = { 1, 2, 4, 8... }. There will be 16 numbers with 5 digits.
- 4. Note the pupils who did not raise their hand.

	••• ••• •••	••	•	•		
16	8	4	2	1		
				•		
0	0	0	0	1	=	1
			•			
0	0	0	1	0	=	2
			•	•		
0	0	0	1	1	=	3
		•• ••				
0	0	1	0	0	II	4
		••		•		
0	0	1	0	1	=	5

# [GUIDE TO USING DOT PAPER TO MAKE BASE 2 NUMBERS]

Lesson Title: Ordering and Comparing Numbers in Base 2	Theme: Number and Numeration		
Lesson Number: M-09-019	Class/Level: JSS 3	Time: 35 minutes	

A-		Λ		a	
	Learning Outcomes	ь Нл	Teaching Aids	A	Preparation
	By the end of the lesson,		None	E.	Write on the board: How
	pupils will be able to:				do you make the base 2
1. List	sets of base 2 numbers			n	umber for 12?
up to 2	20 in ascending order.				
2. Con	npare base 2 numbers up				
to 20.					

- 1. Ask: How do you make the base 2 number for 12?
- Have a pupil from the front of the classroom volunteer to show how to make 12 in base 2. (Answer: As shown).

1	6	8	4	2	1		
		1	1	0	0	=	12

- 3. Ask: How do you read this number?
- 4. Have a pupil from the back of the classroom volunteer to answer. (Answer: one one zero zero.)
- 5. **Say**: Today we are going to learn how to list sets of base 2 numbers. We will also learn how to compare base 2 numbers up to 20.

# Introduction to the New Material (15 minutes)

- 1. Write on the board: { 11, 10 }.
- 2. **Say**: Look at the set of base 2 numbers on the board. How would you work out which number was the smaller one?
- 3. Guide the pupils to say that the number 10 is smaller than the number 11 because when counting, 0 comes before 1.
- 4. **Say**: We can use the symbol < which means "is less than" to compare the 2 numbers.
- 5. Write on the board 10 < 11.
- 6. **Say:** We use the symbol > if one number "is greater than" another.
- 7. Write on the board: { 110, 111, 101 100 }
- 8. Ask: How would we put this set of base 2 numbers in ascending order?
- 9. Discuss as a class how best to work out how to put the numbers in ascending order. (Discussion point: Take the digits one at a time and compare them with each other).
- 10. **Say**: Let us compare the digits step by step. All the numbers start with 1, so let us look at the second digit.
- 11. Ask: What numbers do we have in the second digit? (Answer: 0 or 1).
- 12. Say: There are 2 numbers with 0 as the second digit. 101 and 100. What should we do?
- 13. Guide pupils to say to we should look at the third digit.
- 14. Say: Look at the third digit of the 2 numbers. Which number is smaller?
- 15. Allow a few moments for pupils to think.
- Have a pupil from the left-hand side of the classroom volunteer to call out the answer. (Answer: 100).
- 17. **Say**: Look at the 2 remaining numbers and use the same method to put them in order. Write down all 4 numbers in ascending order.
- 18. Allow the pupils 2 minutes to work in pairs to put the numbers in order.

- 19. Have a pupil volunteer to read out the full set of numbers in ascending order. (Answer: { 100, 101, 110, 111 }).
- 20. Remind pupils to read each digit individually, i.e. 100 is one zero zero.

# **Guided Practice** (5 minutes)

- 1. Write on the board:
  - Put < or > to make the following statements true

a. 1101 🗌 1001 b. 10011 🗌 10010

- 2. Ask pupils to work in pairs to answer the question.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from the front and back of the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers on the board. (Answers: a. 1101 > 1001; b. 10011 > 10010).

# **Independent Practice** (10 minutes)

- 1. Write the following on the board:
  - a. Put the set of base 2 numbers in ascending order { 1101, 10110, 101, 10100, 111 }
  - b. Put the set of base 2 numbers in descending order { 110, 1001, 10100, 1110 }
- 2. Ask the pupils to work independently to answer the questions.
- 3. Walk around, if possible, to check answers and clear misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers on the board. (Answers: a. { 101, 111, 1101, 10100, 10110 }, b. { 10100, 1110, 1001, 110 }).

- 1. Ask: How do we compare base 2 numbers?
- 2. Have a pupil from the right-hand side of the classroom volunteer to give the answer. (Answer: We should take the digits of the numbers one at a time and compare them with each other).

Lesson Title: Converting between Base 10 and Base 2	Theme: Number and Numeration		
Lesson Number: M-09-020	Class/Level: JSS 3	Time: 35 minutes	

Lea	arning Outcomes	ĥ	Teaching Aids	A	Preparation
Ву	the end of the lesson,		None	(Z	Write the table in Step 19
pu	pils will be able to				of the Introduction to
convert nu	umbers up to 50 from			New	Material section on the
base 10 to	base 2 and vice			board	d.
versa.					

- 1. Write on the board: Which is smaller: 10011 or 10001?
- 2. Say: Please raise your hand when you have the answer.
- 3. Wait a few moments or until most of the pupils have their hands raised.
- 4. Have a pupil from the back of the classroom volunteer to call out their answer. (Answer: 10001).
- 5. **Say**: Today we are going to convert numbers up to 50 from base 10 to base 2 and from base 2 to base 10.

# Introduction to the New Material (15 minutes)

- 1. Ask pupils to look at their table from the last lesson for the base 2 numbers from 1 to 20.
- 2. **Say**: The table has powers of 2 going up to 16. What is the next power of 2 we must add to the table to make larger numbers? Raise your hand. (Answer: 32).
- 3. Ask the pupils to add 32 to their tables.
- 4. Write on the board: Convert  $34_{ten}$  to base 2.
- 5. Point to the number  $34_{ten}$ .
- 6. **Say**: When we are working with different bases, it is good practice to write the base of the numbers as shown. This tells us the base of the number we are working with. When we use the table, please remember to write 1 whenever we use a power of 2 to make our number. We can write 0 when we do not use a power of 2.
- 7. Have a pupil volunteer to show how to do this for  $34_{ten}$  on the board. (Answer: As shown below).
- 8. Say: We can also convert from base 10 to base 2.
- Write on the board: Convert 110010<sub>2</sub> to base 10.
- 10. **Say**: We will start by filling in the base 2 number in the table from right to left.
- 11. Complete the table.
- 12. **Say:** Next we will add the powers of base 2 wherever we have a 1.
- 13. Write on the board:  $32 + 16 + 2 = 50_{ten}$
- 14. Allow the pupils a few moments to write the information on the board into their exercise books.
- 15. **Ask**: Who remembers where binary or base 2 numbers are used?
- 16. Guide the pupils to answer that they are used in mobile phones, DVDs and other similar devices.
- 17. **Say**: If we want to send a text message to a friend, the mobile phone converts the message to 1s and 0s. It sends these numbers very quickly so that a message is received almost as soon as it is sent. Let us see how it uses binary numbers to do so.
- 18. Say: We will start by using the numbers 1 to 26 for the letters of the alphabet.

Γ	32	16	8	4	2	1		
ſ	1	0	0	0	1	0	=	34 <sub>ten</sub>
		write as 1000102						
				ne us	10001	.02		
_	32	16	8	<b>4</b>	2	1		
	<b>32</b> 1	<b>16</b> 1	<b>8</b> 0	<b>4</b> 0	<b>2</b> 1	<b>1</b> 0	=	50 <sub>ten</sub>

19. Point to the table below on the board. Ask the pupils to copy it in their exercise books. Note: the numbers have been written without the base<sub>ten</sub>.

а	b	С	d	е	f	g	h	i	j	k	1	т
1	2	3	4	5	6	7	8	9	10	11	12	13
n	0	р	q	r	5	t	и	ν	w	x	У	Ζ
14	15	16	17	18	19	20	21	22	23	24	25	26

20. **Say**: We are going to do it a little easier than a mobile phone would. The basic method is still the same.

## Guided Practice (5 minutes)

- 1. Write on the board: Convert the text "Call me" to base 2
- 2. Guide the pupils to use the table to first assign a base 10 number to each letter of the text.

С	а	Ι	Ι	m	е
3	1	12	12	13	5

- 3. Ask the pupils to work in pairs. They should each convert 3 of the base 10 numbers to base 2. (Both pupils are to convert the letter I).
- 4. Ask pupils to check each other's conversions.
- 5. Walk around, if possible, to check answers and clear misconceptions.
- 6. Have pupils from around the classroom volunteer to give their answers.
- 7. **Write** the correct answers on the board. (Answers: c- 11; a- 1; l- 1100; m- 1101; e- 101.

The complete text is: 11 1 1100 1100 1101 101).

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions below.
- 2. Write the following on the board:

Write the following words in base 2

- a. your name
- b. the day of the week you were born
- c. the month you were born
- 3. Say: You will have to convert any letter you use from 21 to 26 to base 2.
- 4. Walk around, if possible, to check the answers and clear up any misconceptions.
- 5. While pupils work , write the complete table with base 2 numbers for each letter, on the board.
- 6. Have pupils from around the classroom volunteer to give their answers to the questions. Check their answers using the table on the board.
- 7. Ask pupils to exchange exercise books and check each other's work using the table on the board. Ask pupils to make any corrections.

- 1. Say: Please convert 44<sub>ten</sub> to base 2. Raise your hand when you are finished.
- 2. Select a pupil who has raised their hand to call out the answer. (Answer: 101100<sub>2</sub>)

Le	etter	а	b	С	d	е	f	g	h	i	j	k	Ι	т
Ba: nui	se 10 mber	1	2	3	4	5	6	7	8	9	10	11	12	13
Ba nui	ise 2 mber	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101
Le	etter	n	0	p	q	r	S	t	и	v	w	x	У	Z
Ba: nui	se 10 mber	14	15	16	17	18	19	20	21	22	23	24	25	26
Ba nui	ise 2 mber	1110	1111	10000	10001	10010	10011	10100	10101	10110	10111	11000	11001	11010

[GUIDE TO BASE 2 NUMBERS FOR THE ALPHABET]

Lesson Title: Capacity and Mass	Theme: Everyday Arithmetic			
Lesson Number: M-09-021		Class/Level:	: JSS 3	Time: 35 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Differentiate between mass and capacity. 2. Solve problems with masses and problems with capacities.	Teaching None	Aids	Preparation 1. Write on the kilogram lite tonne pint grag gallon poun millilitre The mass of an object matter it contains. The amount of space the 2. Write the question Practice section on the 3. Write the question Independent Practices board.	e board: re am d t is the amount of he volume is the object takes up. is in the Guided he board. hs in the e section on the

 Say: Look at the words on the board. Please copy them in your exercise books. Underline the units for mass and circle the units for

capacity. Raise your hand when you finish.

- 2. Allow 1 minute for the activity. Select a pupil who has raised their hand to show the answer on the board. (Answer: As shown).
- 3. **Say**: Today we are going to learn how to differentiate between mass and capacity. We will also solve problems with both mass and capacity.

# Introduction to the New Material (10 minutes)



- Say: Imagine you have 2 bottles. Each takes 1 litre of liquid. One is full of water, the other one is full of palm oil. One litre of water has a mass of 1000 g. One litre of palm oil has a mass of 890 g. The mass of an object is the amount of matter it contains. The volume is the amount of space the object takes up.
- 2. Ask pupils to copy this information already on the board into their exercise books.
- 3. Say: We use the metric system of measurement for mass and capacity.
- 4. **Ask**: Which units on the board are the metric ones for mass. Which ones are those for volume? Raise your hand.
- 5. Select a pupil who has raised their hand. (Answer: Mass gram, kilogram, tonne; volume litre, millilitre).
- 6. Say: In problems involving mass and volume, we may need to convert between units.

7. Write on the board:

# 1 kg = 1000 g

# 1 litre = 1000 ml

- 8. Say: We will now look at problems with both mass and volume.
- 9. Write a question on mass on the board:
  - Morlai buys a 15 kg bag of rice for a party. He uses 13.5 kg for the party. How much rice does he have left: a. in kg b. in g?
- 10. Ask: What do we need to do in this problem?
- 11. Allow pupils to think about and write down their ideas for 1 minute.
- 12. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
- 13. Ask: Who would like to share their ideas with the class? Raise your hand.
- 14. Select a pupil who has raised their hand. Ask them to explain their method to the class. (Answer: a. [Subtract the 2 masses] 15 kg – 13.5 kg = 1.5 kg [answer in kg];
  - b. [convert kg to g by multiplying by 1000] 1.5 kg  $\times$  1000 = 1500 g [answer in g])
- 15. Write a question on capacity on the board:
  - A bottle contains 2 litres of ginger beer. Adama drinks 350 ml of the ginger beer. Foday drinks 550 ml. How much of the ginger beer is left? Give your answer in:
    - a. ml b. litres
- 16. Allow pupils to think and pair up as before. Have a pupil volunteer to share their ideas with the class.

(Answer:

[add the 2 capacities] 350 ml + 550 ml = 900 ml [amount drunk by Adama and Foday];

[convert litres to ml by multiplying by 1000] 2 litres  $\times$  1000 = 2000 ml;

[subtract amount drunk from the original amount] a. 2000 ml – 900 ml = 1100 ml [answer in ml];

b. [convert answer to litres by dividing by 1000] =  $1100 \text{ ml} \div 1000$  = 1.1 litres [answer in litres]).

# **Guided Practice** (10 minutes)

- 1. Point to the questions on the board:
  - a. A baker needs 125 g of flour to make a loaf of bread. How much flour does he need to make 12 loaves? Please give your answer in kg.
  - b. A bottle contains 3 litres of water. It is used to fill glasses which hold 150 ml each. How many glasses does it fill?
- 2. Walk around, if possible, to check the answers and clear up any misconceptions.
- 3. Have pupils from around the classroom volunteer to give their answers to the questions.
- 4. Write the correct answers with the calculation on the board. (Answers: a.  $125 \times 12 = 1500 \text{ g} = 1.5 \text{ kg}$ ; b. 3 litres =  $3 \times 1000 = 3000 \text{ ml}$ ,  $\frac{3000}{150} = 20 \text{ glasses}$ ).

# **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the following questions on the board:
  - a. A can containing 5 litres of palm oil has a mass of 4.45 kg. It is used to fill bottles which hold 1.25 litres of palm-oil each.
    - i. How many bottles of palm-oil does it fill?
    - ii. What is the mass of each bottle of palm-oil? Please give your answer in g.
  - b. A bag contains 3.5 kg of rice. A second bag contains 3 times as much.
    - i. How much rice does the second bag contain?
      - ii. What is the difference in mass between the 2 bags?

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers with the calculation on the board. (Answers: a. i.  $\frac{5}{1.25} = \frac{500}{125} = 4$

bottles, ii. 4.45 kg = 4450 g,  $\frac{4450}{4}$  = 1112.5 g; b. i. 3.5 × 3 = 10.5 kg, ii. 10.5 – 3.5 = 7 kg).

- 1. Ask: Who can tell the class what mass is? Raise your hand.
- 2. Select a pupil who has raised their hand to answer. (Answer: Mass is the amount of matter an object contains.)
- 3. Ask: Who can tell the class what volume is? Raise your hand.
- 4. Select a different pupil who has raised their hand to answer. (Answer: The amount of space an object takes up.)

Lesson Title: Percentages of Quantities	Theme: Everyday Arithmetic	
Lesson Number: M-09-022	Class/Level: JSS 3	Time: 35 minutes

	Learning Outcomes	N AA	Teaching Aids		<b>Preparation</b>
$((\bigcirc))$	By the end of the lesson,		None	(	🕺 Write the question in the
	pupils will be able to	ØS		E	Independent Practice on
find pe	ercentages of quantities.			t	he board.

- 1. **Say:** Percentages are used all the time in everyday life. Please give me one example where percentages are used. Raise your hand. (Example answer: To find by how much a quantity has increased or decreased)
- 2. **Say**: When we write a percentage, e.g. 60%, we mean 60 parts out of 100. We write this as  $\frac{60}{100}$ . This means that if we divide the whole of a quantity into 100 parts, then we are interested in 60 of the parts. Today we are going to calculate percentages of quantities.

# Introduction to the New Material (14 minutes)

- 1. Write on the board: Find 60% of 150 kg.
- 2. **Say**: We want to find out if we divide 150 kg into 100 equal parts, how many kg will 60 of the parts be equal to?
- 3. Show on the board how the above calculation is done:

$$60\% \text{ of } 150$$

$$= \frac{60}{100} \times 150$$

$$= \frac{60 \times 150}{100}$$

$$= 6 \times 15$$

$$= 90 \text{ kg}$$

- 4. **Say**: Always simplify the calculation by cancelling zeros or dividing by a common factor.
- 5. Write on the board: Find 25% of 80 m.
- 6. **Say**: Now we want to find out if we divide 80 m into 100 equal parts, how many m will 25 of the parts be equal to?
- 7. Guide a pupil to show the calculation on the board:

25% of 80

$$= \frac{25}{100} \times 80$$

$$= \frac{25 \times 80}{100}$$
cancel the 0s on the numerator and denominator,  
divide by common factor 2
$$= \frac{25 \times 4}{5}$$

$$= 20 \text{ m}$$

8. **Say**: Percentages are found everywhere in everyday life.

- 9. Write on the board: A farmer owns 360 acres of land. He plants rice on 30% of his farm. How many acres of land are planted with rice?
- 10. Ask: What calculation should we do for this problem? Give pupils 1 minute to think and then raise their hand to answer. (Answer: 30% of 360).
- 11. Have a pupil volunteer to show the calculation on the board. (Answer: 30% of 360 =  $\frac{30}{100} \times 360$  =

 $\frac{30 \times 360}{100} = 3 \times 36 = 108 \text{ acres})$ 

#### **Guided Practice** (5 minutes)

- 1. Write on the board: Fatu scored 75% on a test. The total marks for the test was 56. How many marks did Fatu score?
- 2. Ask: What calculation should we do for this problem? (Answer: 75% of 56).
- 3. Ask pupils to work in pairs to solve the problem.
- 4. Walk around, if possible, to check the answers and clear up any misconceptions.
- 5. Have a pupil volunteer to show the calculation on the board.
- 6. Correct any errors on the pupil's calculation on the board. Ask pupils to correct their work. (Answer: 75% of 56 =  $\frac{75}{100} \times 56 = \frac{75 \times 56}{100} = \frac{3 \times 56}{4} = 42$  marks).

## **Independent Practice** (10 minutes)

- Point to the following problems on the board: 1.
  - a. There are 250 JSS 3 pupils in a school. 40% of them are girls. How many are girls?
  - b. Aisha had Le 150,000. She spent 25% of it on books. How much did she have left?
  - c. Kemi and Yemi received Le 50,000 from their grandfather. Kemi's share of the money was 45%. How much did each receive?
- 2. Ask the pupils to work independently to solve the problems.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Select pupils from around the classroom to give their answers to the questions.
- 5. Correct any errors. (Answers: a. 40% of  $250 = \frac{40}{100} \times 250 = \frac{40 \times 250}{100} = 4 \times 25 = 100$  girls; b. 25% of 150,000 =  $\frac{25}{100} \times 150,000 = \frac{25 \times 150}{100} = \frac{1 \times 150}{4} = 37,500,$ 
  - Aisha had Le 150,000 37,500 = Le 112,500 left;
  - Kemi's share: 45% of 50,000 =  $\frac{45}{100} \times 50000 = 45 \times 500 = Le 22,500$ , c. Yemi's share was Le 50,000 - 22,500 = Le 27,500).

- 1. Ask: How do we simplify a percentage calculation? Raise your hand.
- 2. Have a pupil from the back of the classroom volunteer to answer. (Answer: By cancelling zeros. Sometimes we also have to divide by a common factor.)

Lesson Title: Percentage Increase and Decrease	Theme: Everyday Arithmetic	
Lesson Number: M-09-023	Class/Level: JSS 3	Time: 35 minutes
	•	

Learning Outcomes By the end of the lesson,	<b>Teaching Aids</b> None	<ul><li>Preparation</li><li>1. Write the following on the board:</li></ul>
pupils will be able to:		a. <u>percentage</u> b. <u>percentage</u>
1. Calculate the percentage		increase given 2 <u>decrease</u> , given 2
increase or decrease, given 2		numbers numbers
numbers. 2. Increase and decrease quantities by a percentage.		c. <u>increasing</u> d. <u>decreasing</u> <u>quantities</u> by a <u>quantities</u> by a percentage percentage
		2. Write the problems in the Independent Practice section on the board

- 1. Ask: Who can tell the class what we did in the last lesson? Raise your hand.
- 2. Have a pupil from the back of the classroom answer. (Answer: Percentages of quantities)
- 3. **Say**: In this lesson, we are going to learn how to calculate the percentage increase or decrease given 2 numbers. We will also do calculations on increasing and decreasing quantities by a percentage.

#### Introduction to the New Material (10 minutes)

- 1. **Say**: When we calculate with percentages, it is important to be clear on the type of calculation the question requires us to do. There are 2 main types of percentage problems shown on the board. One deals with percentage increase and decrease given 2 numbers. The other type deals with increasing or decreasing quantities by a given percentage. We will go through each type of problem step by step.
- 2. Write Question a. on the board: The price of a pen increased from Le 1,000 to Le 1,200. What is the percentage increase?
- 3. Remind pupils of the formula used for a percentage increase given 2 numbers.
- 4. Explain each step of the problem. See the problem worked out in the example below.
- 5. At the point marked \*, pause for a moment. **Ask:** What do we do next?
- 6. Guide a pupil to give the next step. (Answer: Substitute the values for the increase and original amounts in the formula.)
- 7. Continue with working out the problem. Calculate the percentage increase.
- 8. **Write** Question b. on the board: The price of a bag of cement decreased from Le 50,000 to Le 45,000. What is the percentage decrease?
- 9. Have a pupil volunteer to solve the problem on the board with your help.

a.	Percentage increase given 2 numbers
The p	rice of a pen increased from Le 1000 to
Le 1,2	00. What is the percentage increase?

Percentage increase	=	$\frac{\text{increase}}{\text{original amount}} \times 100$
Increase	=	1,200 - 1,000
	=	200

b. <u>Percentage decrease given 2 numbers</u> The price of a bag of cement decreased from Le 50,000 to Le 45,000. What is the percentage decrease?

Percentage decrease	=	$\frac{\text{decrease}}{\text{original amount}} \times 100$
Decrease	=	50,000 - 45,000
	=	5,000

*Percentage increase	=	$\frac{200}{1,000} \times 100$	Percentage decrease	=	$\frac{5,000}{50,000} \times 100$
	=	$\frac{200\times100}{1,000}$		=	$\frac{5,000\times100}{50,000}$
	=	20%		=	10%

- 10. Check that pupils recognise that to calculate both an increase and decrease, they always have to subtract the higher number from the lower number.
- 11. If pupils make a mistake, help them make the proper corrections.
- 12. Say: Now let us look at the other type of percentage problems.
- 13. Write Question c. on the board: Increase 40 kg by 50%.
- 14. Ask: Will our answer be in % or kg?
- 15. Give pupils a moment to think.
- 16. Have a pupil volunteer to answer the question. (Answer: 'kg')
- 17. **Say**: In Question c., we need to increase a quantity. This quantity can be a measurement in metres, kilograms, Leones, etc. The answer will be in the same unit.
- 18. Ask: How is this different from the previous 2 questions we just answered?
- 19. Help pupils understand that questions a. and b. were on increasing or decreasing 'percentages' so the answers were also in percentages.

This question is about increasing a 'quantity.' The answer will be in the unit of measurement in the question.

- 20. **Say:** Maybe you already learned a way to solve increasing or decreasing quantities. We will look at 2 different methods and you can choose the one you prefer.
- 21. Solve the problem using 2 different methods (as shown below). Explain each step as you do it.

с.	<u>Increase qua</u>	ntit	ies by a percentage				
In	crease 40 kg by	/ 5%	6				
M	<u>ethod 1:</u>			Method 2:			
5%	6 of 40 kg	=	$\frac{5}{100} \times 40$	100% + 5%	=	105%	
		=	$\frac{5 \times 40}{100}$	New	=	$105 \times 40$	
		=	2 kg	quantity		$\frac{100}{100}$ × 40	
Ne	ew quantity	=	40 + 2		=	$\frac{105\times40}{100}$	or $1.05 \times 40$
		=	42 kg		=	42 kg	

22. **Say**: Either of these methods can be used to find the new quantity after a percentage decrease.

#### Guided Practice (10 minutes)

- 1. Point on the board to d. Decrease quantities by a percentage
- 2. Write this question: Decrease Le 60,000 by 10%
- 3. Ask the class to work in pairs.
- 4. Say: This question asks us to decrease the quantity. What calculation should we use?
- 5. Give pupils time to think about the question then have a pupil volunteer to answer. (Answer: Subtraction.)
- 6. **Say**: In your pairs, one of you will use Method 1 and the other will use Method 2 to calculate the answer. Choose which one you want to use now.
- 7. Say: Please raise your hand when you finish.
- 8. Allow pupils 2 minutes to answer the question.
- 9. Walk around, if possible, to check the answers and clear up any misconceptions.

- 10. Ask for one pupil who chose Method 1 and another who chose Method 2 to volunteer to come to the board.
- 11. Ask the pupils at the board to write down their solutions.
- 12. Whilst they are doing this, ask the rest of the class for each pair to show their partner their calculation.
- 13. **Say**: Please show your partner the calculation you did to find the answer. Discuss your methods together.
- 14. Allow a further minute for the pupils to discuss and share ideas.
- 15. Ask: Do we agree with the calculations on the board?
- 16. Ask for views from several pupils.
- 17.Correct any errors on the calculation on the board. Ask pupils to correct their work. (Answer: Method 1:  $\frac{10}{100} \times 60,000 = \frac{10 \times 60,000}{100} = 6,000$ , new amount = 60,000 - 6,000 = Le 54,000; Method 2: 100% - 10% = 90% = 0.9, new amount = 0.9 × 60,000 = Le 54,000).
- 18. **Say:** Most people prefer Method 2 because you can do the calculation for the increase or decrease in your head, and easily divide by 100. You can then simply multiply that figure by the original quantity to get the new quantity.

# **Independent Practice** (10 minutes)

- 1. Point to the following problems on the board:
  - a. The number of pupils in a Junior Secondary School increased from 820 to 861 Calculate the percentage increase
  - b. Alusine's tax decreased from Le 6,000 to Le 3,900 Calculate the percentage decrease
  - c. Increase Le 150,000 by 8%
  - d. Decrease 900 m by 30%
- 2. Ask the pupils to work independently to answer the questions. They can choose method 1 or method 2 for questions c. and d.
- 3. Walk around, if possible, to check the answers and correct any mistakes.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers on the board.
  - (Answers:
  - a. Increase = 861 820 = 41, percentage increase =  $\frac{41}{820} \times 100 = 5\%$ ;
  - b. Decrease = 6,000 3,900 = 2,100; percentage decrease =  $\frac{2100}{6000} \times 100 = 35\%$ .
  - c. Method 1:  $\frac{8}{100} \times 150,000 = \frac{8 \times 150,000}{100} = 12,000$ , new amount = 150,000 + 12,000 = Le 162,000; Method 2: 100% + 8% = 108% = 1.08, new amount = 1.08 × 150,000 = Le 162,000.
  - d. Method 1:  $\frac{30}{100} \times 900 = \frac{30 \times 900}{100} = 270$ , new amount = 900 270 = 630 m; Method 2: 100% - 30% = 70% = 0.7, new amount = 0.7 × 900 = L630 m).

- 1. Ask pupils what they learned during the lesson.
- 2. Allow pupils to raise their hand and give their views. (Example answers: Different calculations for solving a percentage increase or decrease from an increase or decrease of a quantity. Accept all reasonable answers.)

Lesson Title: Ratio	Theme: Everyday Arithmetic	
Lesson Number: M-09-024	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	F Teaching Aids	Preparation
By the end of the	None	1. Write on the board:
lesson, pupils will be		A JSS 3 class has 35 girls and 30
able to:		boys. Find:
1. Review the forms of ratio: m :		a. The ratio of girls to boys
n and $\stackrel{\mathrm{m}}{-}$ .		b. The ratio of boys to girls
n 2 Divide a number into a given		Please give your answers in the lowest
z. Divide a number into a given		terms.
3 Solve ratio problems and		2. Write the questions from the Guided
simplify answers to the lowest		Practice section on the board.
terms		3. Write the questions from the
		Independent Practice section on the
		board.

- 1. Say: Discuss with your seatmate everything you know about ratios.
- 2. Allow the pupils to work in pairs for 2 minutes to discuss all they remember about ratios.
- Have 2-3 pupils volunteer to tell the class what they know. Accept all reasonable answers. (Example answers: A ratio is used to compare 2 or more quantities. The quantities must be measured in the same unit. We can write a ratio using a colon, :, or as a fraction).
- 4. **Say**: Today we are going to review the forms of ratios, and how to divide a number into a given ratio. We will also solve problems and simplify our answers to the lowest terms.

# Introduction to the New Material (10 minutes)

- 1. **Say**: We use ratios to compare quantities of the same type, e.g. length, weight, people, etc. We change the units if they are different e.g. metres to centimetres, or grams to kilograms. We describe ratios in 2 different ways. In a JSS 3 class, there are 24 girls and 36 boys.
- 2. Write on the board: 24 girls is to 36 boys.
- 3. Ask: How do we write this as a ratio? Raise your hand. (Answer: 24 : 36)
- 4. Ask: What other way can we write this ratio? Raise your hand. (Answer: as a fraction,  $\frac{24}{26}$ )
- 5. **Say:** It does not matter which of the 2 ways we write a ratio, we should always simplify it to its lowest terms. Simplify the ratio on the board by dividing with the common factor, **12**.
- 6. Give pupils 1 minute to work out the answer in their exercise.
- 7. Have one pupils volunteer share the answer and explain what it means. (Answer: 2 : 3,  $\frac{2}{3}$  . This means that for every 2 girls there are 3 boys).
- 8. **Say**: It is very important to keep the order of the ratio as given in the problem. A ratio written as  $2:3 \text{ means } \frac{2}{3}$  while a ratio written as  $3:2 \text{ means } \frac{3}{2}$ . The fractions are different and give different answers. A ratio compares 2 quantities. If we know the ratio and the total number of quantities, we can calculate how many of one quantity we have compared to the other.
- 9. Write on the board: Share 75 kg in the ratio 2 : 3
- 10. Say: Use any method you know to solve the problem.
- 11. Allow 1 minute for pupils to answer the question.
- 12. Have a pupil volunteer to come to the board. Ask them to explain their steps to the class.

13. It is possible to solve the problem in several ways. One possibility is shown below.

 $2+3 = 5 \qquad \qquad \text{Work out the total number of shares}$   $\frac{75}{5} \times 2 = 15 \times 2 = 30 \text{ kg} \qquad \qquad \text{Work out the quantity according to its share (2)}$   $\frac{75}{5} \times 3 = 15 \times 3 = 45 \text{ kg} \qquad \qquad \text{Work out the quantity according its share (3)}$ 

- 14. Check if others in the class have used a different method from the one on the board.
- 15. Check that the methods give the same answer. Tell them to use the method they prefer.
- 16. Say: We can check our calculations by adding the 2 quantities to see if we get the original amount. You will know that you made a mistake if you get a different amount.
- 17. Write on the board: 30 kg + 40 kg = 75 kg
- 18. Say: We know our calculation is correct as we get the amount we started with. We can also solve problems with ratios. We will compare the girls and boys in the class.
- 19. Refer to the problem already written on the board. You can replace the numbers in the problem by those from the actual counts of boys and girls in the class.
- 20. Say: We are going to count the number of girls and boys in the class.
- 21. Ask all the girls to stand up. Ask a boy to count the girls. Write the number in the problem.
- 22. Ask all the boys to stand up. Ask a girl to count the boys. Write the number in the problem.
- 23. Allow 1 minute for pupils to answer the questions.
- 24. Have 2 pupils volunteer to come to the board at the same time. Ask each to show how they write one of the ratios.
- 25. Use the actual numbers for your JSS 3 class. Depending on the numbers, you may not be able to simplify your answers. (Example answers: 35:30 = 7:6;30:35 = 6:7).

#### **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Point to the guestions on the board:
  - a. Share 48 litres in the ratio 3 : 5. Please check your answer.
  - b. The shape of a room is a rectangle. One side is 3.5 m long and the other is 5 m Find: i. the ratio of width to length ii. the ratio of length to width Give your answers in the lowest term.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Allow pupils to call out the answers.
- 5. Write the correct answers on the board. Ask pupils to check their work. (Answers: a.  $\frac{48}{8} \times 3 =$

 $6 \times 3 = 18$  litres;  $\frac{48}{8} \times 5 = 6 \times 5 = 30$  litres,

Check: 18 litres + 30 litres = 48 litres; b. i. 3.5 : 5 = 7 : 10, ii. 5 : 3.5 = 10 : 7)

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to continue to work independently to answer the questions.
- 2. Point to the questions on the board:
  - a. Share 450 m in the ratio 5 : 4. Please check your answer.
  - b. A Junior Secondary School has 7 teachers and 420 pupils. Find: i. the ratio of pupils to teachers ii. the ratio of teachers to pupils Give your answers in the lowest term.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.

5. Write the correct answers on the board. Ask pupils to check their work. (Answers: a.  $\frac{450}{9} \times 5 = 50 \times 5 = 250$  m;  $\frac{450}{9} \times 4 = 50 \times 4 = 200$  m, Check 250 m + 200 m = 450 m; b. i. 420 : 7 = 60 : 1, ii. 7 : 420 = 1 : 60)

- 1. Say: Please tell the class something you learned today about ratios. Raise your hand.
- 2. Allow a pupil one minute to explain. (Example answer: How to write ratios using a colon; how to use a fraction to write a ratio; sharing an amount in a given ratio; solve ratio problems.)
- 3. **Say**: Excellent work class! In the next lesson, we will learn about a special type of ratio called 'rates'.

Lesson Title: Rates	Theme: Everyday Arithmetic	
Lesson Number: M-09-025	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	Preparation
By the end of the lesson,	None	1. Write on the board:
pupils will be able to:		Write the ratio 250 cm to 2m in its
1. Identify that rate is a special		lowest terms.
ratio that compares 2 units of		2. Write the problem from the Guided
measurement.		Practice section on the board.
2. Solve problems involving rate.		3. Write the problems from the
		Independent Practice section on the board.

- 1. Ask pupils to solve the ratio problem on the board. Allow 1 minute.
- 2. Have a pupil volunteer to come to the board and explain how to solve the problem.
- 3. Ask the class if they agree. Discuss and correct any misconceptions.
- 4. Write the answer with the steps on the board.



5. **Say**: Today we are going to identify that rate is a special ratio that compares 2 units of measurement. We will also solve problems involving rate.

# Introduction to the New Material (10 minutes)

- 1. **Ask**: Who can tell the class what we did just now before comparing the 2 quantities? Raise your hand.
- 2. Guide a pupil to think about the first step in the calculation above. (Answer: The quantities were changed to the same unit.)
- 3. **Say**: Sometimes we want to compare 2 quantities that are not alike and cannot be changed to the same unit. An example is when we want to work out how fast a car is travelling.
- 4. Ask: Who can give the 2 quantities we compare?
- 5. Guide the pupils to think of the formula and units of speed. (Answer: Distance and time, metres per second, kilometres per hour.)
- 6. **Say**: When we compare quantities that are 2 different units of measurement we use a special type of ratio called a rate. Let us see if we can tell which of the following is a ratio and which is a rate. Think about the units of measurement of the quantities.
- 7. Allow a few moments after each statement for pupils to work in pairs to discuss and share their ideas. **Write** each answer on the board.
  - a. **Ask:** What is the length of a rectangle compared to its width? Raise your hand. (Answer: Ratio, units of length, e.g. m, cm)
  - b. **Ask**: What is the number of metres you travel in one second? Raise your hand. (Answer: Rate, units are metres and second (metres per second))
  - c. **Ask**: What is the area of a square compared to the area of a triangle? Raise your hand. (Answer: Ratio, units of area, e.g. cm<sup>2</sup>, m<sup>2</sup>)
  - d. **Ask:** What is the percentage of interest you pay in one year? Raise your hand. (Answer: Rate, actually called 'interest rate', units are percentage and year (percentage per annum))

- 8. **Ask:** What do you notice about the answers on the board? Raise your hand.
- Guide pupils to look at the units of measurement of the quantities.
   (Example answer: The quantities in the ratio are measured with one unit, the quantities in the rate are measured with 2 units.)
- 10. Say: The units in a ratio cancel each other because they are the same. The units in a rate take on the unit from the numerator and the unit from the denominator, e.g.  $\frac{\text{km}}{\text{hr}}$ .
- 11. Write on the board: A car travels a distance of 240 km in 3 hours. What is the average speed in kilometres per hour (km/hr)?
- 12. Show how to solve this problem on the board. Explain each step as shown.



- 13. **Say**: The speed in our example is the unit rate at which the car is travelling. A unit rate is a ratio that tells us how many units of one quantity there are for every one unit of the second quantity.
- 14. **Write** the definition of the unit rate on the board: A ratio that tells us how many units of one quantity there are for every one unit of the second quantity.
- 15. **Say**: In our example, this is '80' kilometres for every '1' hour, i.e. 80 km/hr. Rates use words and symbols such as 'per' (/), 'each' (ea) and 'at' (@).
- 16. Write: Rates use words and symbols such as 'per' (/), 'each' (ea) and 'at' (@).

# **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Point to the question on the board: Mabinti spends Le 20,000 to buy 4 pints of palm oil. How much did it cost her per pint?
- 3. Walk around, if possible, to check answers and clear misconceptions.
- 4. Have a pupil volunteer to come to the board and explain how to solve the problem.
- 5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer:  $\frac{20000}{4}$  = Le 5,000/pint)

# **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:
  - a. A bunch of 4 plantains costs Le 12,000. How much does it cost per plantain?
  - b. A car travels at 560 km in 7 hours. What is its speed?
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have a pupil volunteer to call out their answer to the problem.
- 6. Write the correct answer on the board. Ask pupils to correct their work. (Answers: a.  $\frac{12000}{4}$  =

Le 3,000/plantain; b. 
$$\frac{560}{7} = 80$$
 km/hr)

- 1. **Ask:** What is unit rate? Raise your hand.
- 2. Have a pupil at the back of the classroom to answer. (Answer: A unit rate is a ratio that tells us how many units of one quantity there are for every one unit of the second quantity.)
- 3. **Say**: In the next lesson, we will learn how to use the unit rate to calculate with direct proportions.

Lesson Title: Direct Proportion	Theme: Everyday Arithmetic	
Lesson Number: M-09-026	Class/Level: JSS 3	Time: 35 minutes

learning Outcomes	Q	Teaching Aids	R Preparation					
By the end of the lesson, pupils will be		None	1. Write the	e foll	owir	ng pr	oble	m on
able to:			A lorry travels at 1	.80 k	m in	3 hc	ours.	What
1. Identify the symbol for			is its average spee	d in	km/	h?		
proportionality ( $\infty$ ), the means			2. Draw the table	belo	w:			
and extremes.			Distance, d (km)			180		
2. Solve direct proportion			Time <i>,</i> t (h)	1	2	3	4	
problems.			3. Write the probl	em i	n the	e Gui	ded	
			Practice section or	n the	e boa	ard.		
			4. Write the probl	em i	n the	e Ind	eper	ndent
			Practice section or	n the	e boa	ard.		

- 1. Point to this problem on the board: A lorry travels at 180 km in 3 hours. What is its average speed in km/h?
- 2. Say: Please look at the problem on the board. You have 2 minutes to solve it.
- 3. Allow pupils to call out the answer. (Answer:  $\frac{180}{3} = 60$  km/h)
- 4. **Say**: Today we are going to identify the symbol for proportionality ( $\infty$ ), the means and extremes. We will also solve problems on direct proportion.

#### Introduction to the New Material (10 minutes)

- 1. Ask: How far does the car in our problem travel after 1 hour? Raise your hand. (Answer: 60 km)
- 2. Ask the pupils to put the information in the table. Ask them to use the information to complete the table for the remaining times. (Answers: As shown.)
- 3. **Ask:** Who can tell the class what they notice in the table? Raise your hand.

Distance, d (km)	60	120	180	240
Time, t (h)	1	2	З	4
Speed (unit rate) (km/h)	60	60	60	60

4. Guide a pupil to explain the relationship between distance and time. (Example answers: When the distance doubles from 60 km to 120 km the time also doubles from 1 hour to

from 60 km to 120 km the time also doubles from 1 hour to 2 hours. The unit rate stays the same – it is always 60 km/h).

- 5. Complete the third row of the table showing that the unit rate is always 60.
- 6. Say: From the table, we can see that all the rates are equal to each other. We can take any 2.
- 7. Write on the board:
- 8. Say: We can write the rates as 60:1 = 120:2

60	_	120
1	-	2
60	=	60

equivalent fractions:

9. **Say**: In any rates (or ratio) problem, where we have equal ratios, a : b = c : d we can write this as equivalent fractions:

$$\frac{a}{b} = \frac{c}{d}$$

- 10. Say: Cross-multiplying gives us:  $a \times d = b \times c$
- 11. **Say**: The quantities have special names, 'a' and 'd' are called 'extremes';

'b' and 'c' are called 'means'.

- 12. Write on the board: The product of the extremes equals the product of the means.
- 13. **Say**: In our example we can write:

$$60 \times 2 = 120 \times 1$$
  
120 = 120

- 14. **Say**: Writing the ratios like this is very useful. In a lot of ratio or rates problem, we usually know 3 of the quantities and we need to find the fourth. We can see how this works in our problem.
- 15. **Say**: Find the distance, x, travelled by the lorry after 6 hours. We will start by writing:

$$60:1 = d:6$$

- 16. Ask the pupils to work in pairs to continue to discuss and solve the problem.
- 17. Have a pupil from the left-hand side of the classroom volunteer to come to the board and explain how to solve the problem.

$\frac{60}{1}$	=	$\frac{d}{6}$	←	Write ratios as equivalent fractions
60 × 6	=	$d \times 1$	←	Cross-multiply (product of extremes = product of means)
360	=	d	←──	Solve for x by multiplying
d	=	360 m	←──	Distance travelled in 6 hours

- 18. **Say**: This shows that when the distance goes up 6 times from 60 km to 360 km, the time also goes up 6 times from 1 hour to 6 hours. This means the distance travelled, d, is directly proportional to the time, t.
- 19. Write on the board:  $d \propto t$  read as d 'is directly proportional to' t
- 20. Say: This can be written as an equation: d = kt where k is called the constant of proportionality
- 21. **Say**: In our example the unit rate, 60 km/h, is the constant of proportionality. It is the constant (steady or average) speed at which the car travels. We can write the equation as:

22. Say: If d and t are known, we can find k using this form of the equation:

 $\frac{d}{t} = k$  k will always be the same no matter the values of d and t

23. Say: We can use this fact to solve similar problems on direct proportionality.


# **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs to solve the problem.
- 2. Point to this problem on the board:

A car travels at a steady speed of 60 km/h. Using the equation from our example, find:

a. the distance, d, it travels after 5 hours. b. the time it takes to travel 360

km

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have a pupil from the right-hand side of the classroom volunteer to explain how to solve the problem on the board.
- 5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a d = 60 t t = 5 hours  $d = 60 \times 5 = 300 \text{ km}$ )

(Answers: a. 
$$u = 00^{\circ}$$
,  $v = 5^{\circ}$  hours,  $u = 00^{\circ} \times 5^{\circ} = 500^{\circ}$  km,

b. d = 60 t, d = 360 m, 360 =  $60 \times t$ , t =  $\frac{360}{60}$  = 6 hours)

### Independent Practice (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the following ratio problem on the board:
  - The distance a car travels, d, is directly proportional to the time, t.
  - If it travels 150 km at a steady speed for 2 hours:
    - a. Find the constant of proportionality, k in km/h.
    - b. Use k to find the time it takes to travel 300 km.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from the front and back of the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work.

(Answers: i. d 
$$\propto$$
 t; d = k t; k =  $\frac{d}{t}$ ; k =  $\frac{150}{2}$  = 75 km/h;

ii. 
$$75 = \frac{300}{t}$$
,  $t = \frac{300}{75}$ ,  $t = 4$  hours)

- 1. Say: Please write your name on a piece of paper. You will hand it in at the end of the lesson.
- 2. **Say:** A quantity, y, is directly proportional to another quantity, x. Write on your paper how you show this as an equation if the constant of proportionality is k.
- 3. Ask the pupils to hand in their papers.
- Go through these papers after the lesson to check their understanding of the topic. Use a similar question at the start of the next lesson to clear up any misconceptions.
  (Answer: y = kx).

Lesson Title: Indirect Proportion	Theme: Everyday Arithmetic	
Lesson Number: M-09-027	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None	1. Write on the board:
lesson, pupils will be		The cost, C, is directly proportional to
able to:		the number, n, of items bought. Write this
1. Identify the form of an		relationship as an equation.
indirectly proportional		2. Write the problems from the Guided
relationship (t $\propto \frac{1}{s}$ ).		Practice section on the board.
2. Solve indirect proportion		Independent Practice section on the board
problems.		

- 1. Point to the following question: The cost, C, is directly proportional to the number, n, of items bought. Write this relationship as an equation.
- 2. Say: Please answer the question on direct proportions on the board. You have 2 minutes.
- 3. Have a pupil volunteer to come to the board to explain how to write the relationship. (Answer: C = kn where k is the constant of proportionality)
- 4. Ask the class if they agree. Discuss any misconceptions for example, some pupils may have written,  $C \propto n$ . Explain that this is the correct expression for the relationship between C and n.
- 5. Explain that an equation always has an equal sign, so C = kn is the correct answer. We must define k if it is not done in the question.
- 6. **Say**: Today we are going to identify the form of an indirectly proportional relationship (t  $\propto \frac{1}{s}$ ). We will also solve problems with indirect proportion.

# Introduction to the New Material (10 minutes)

- 1. Say: In direct proportion problems, when one quantity gets bigger, the other quantity also gets bigger. We see we can write this as either a ratio (C : n) or as a formula using a constant of proportionality (C = kn).
- 2. Write: (C : n) or (C = kn).
- 3. **Say**: There are other times when one quantity gets smaller as another gets bigger. Think about a situation when a car is travelling at different speeds. It will take the car less time to cover a particular distance if it travels fast than if it travels slowly.
- 4. Say: Suppose we are going on a 200 km journey.
- 5. Draw this table on the board. Complete the first 2 rows.

Speed, s (km/h)	50	40	25
Time, t (hours)	4	5	8
Constant of proportionality, k			

6. Ask: Who can tell the class what they notice in the table? Raise your hand.

- 7. Guide a pupil to explain the relationship between the speed and time. (Example answers: When the speed halves from 50 km/h to 25 km/h the time doubles from 4 hours to 8 hours. When you multiply any speed by its time we get 200.)
- 8. Say: We can see from the table that multiplying each speed by its related time gives the same answer, 200. We can also see that as the speed increases, the time decreases. This means the time it takes to travel, t, is 'indirectly proportional' to the speed, s.
- 9. Write on the board:

- 10. **Say**: This can be written as an equation:
- t  $\propto \frac{1}{s}$  read as t 'is indirectly proportional to's t =  $\frac{k}{s}$  where k is called the constant of proportionality.
- 11. Therefore, the constant of proportionality,  $k = s \times t = 200$
- 12. Ask: What is the unit of the constant of proportionality? Raise your hand. (Answer: km)
- 13. Say: This is the distance for the journey. It remains constant.
- 14. Ask pupils to use the information to complete the row for the constant of proportionality.
- 15. Say: In our example we can write the equation as:

$$t = \frac{200}{s}$$

- 16. Ask: How long does it take the car to travel at a speed of 20 km/h?
- 17. Allow the pupils to work in pairs for 2 minutes to solve the problem.
- 18. Have a pupil volunteer to come to the board to explain how to solve the problem.

t = 
$$\frac{200}{s}$$
 equation for relationship, k = 200 km  
=  $\frac{200}{20}$  substitute 20 km/h for speed  
errors on the = 10 hours time taken at 20 km/h  
calculation on  
the board. Ask  
pupils to check their work.

- 20. Say: If s and t are known, we can find k using this form of the equation:  $s \ge t = k \cdot k$  will always be the same no matter the values of s and t.
- 21. **Say**: We can use this fact to solve similar problems on indirect proportionality.

# **Guided Practice** (10 minutes)

19.

- 1. Ask pupils to work in pairs to solve the problem.
- 2. Point to the question on the board:

A car travels a distance of 300 km at a steady speed of 50 km/h. How long does it take to complete the journey?

- 3. Walk around, if possible, to check answers and clear misconceptions.
- 4. Have a pupil from the left-hand side of the classroom volunteer to give their answer to the questions.
- 5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: k = 300 km, s = 50 km/h, t =  $\frac{k}{s} = \frac{300}{50} = 6$  hours ).

# Independent Practice (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:
  - The time t, it takes a van to travel on a journey is indirectly proportional to its speed, s.
  - It takes a van 3 hours to complete its journey travelling at a steady speed of 50 km/h.
    - a. Find the constant of proportionality, k in km.
    - b. Use k to find the time it takes if it travels at a steady speed of 75 km/h.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from the front and back of the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: i. t  $\propto \frac{1}{s}$ , t =  $\frac{k}{s}$ , k = t s = 3 × 50 = 150 km; ii. t =  $\frac{150}{75}$  = 2 hours).

- 1. Allow a few moments after each question for pupils to answer. Write the answers on the board.
  - a. Say: A quantity, y, is directly proportional to another quantity, x.
     Write how to show this as an equation if the constant of proportionality is k. (Answer: y = kx).
  - b. Say: A quantity, y, is indirectly proportional to another quantity, x. Write how to show this as an equation if the constant of proportionality is k. (Answer:  $y = \frac{k}{x}$ ).
- 2. **Say**: In our next lesson, we will use both these equations to answer problems on direct and indirect proportionality.

Lesson Title: Proportion Problem Solving Theme: Everyday Arithmetic				
Lesson Number: M-09-028		Class	s/Level: JSS 3	Time: 35 minutes
Learning Outcomes By the end of the lesson, pupils will be able to solve direct and indirect proportion problems.	Teaching None	Aids	<ul> <li>Preparation <ol> <li>Write the followin board:</li> <li>The cost, C, of putting a directly proportional to roof. A roof costs</li> <li>Le 18,000,000 and cove i. Find the relationsh ii. Find out how much roof with an area of iii. A roof costs Le 15, does it cover?</li> <li>The time, T, it takes to proportional to the nur digging the well. It takes dig a well. <ol> <li>Find the relationsh ii. Find out how long working at the sam well.</li> </ol> </li> </ol></li></ul>	ng problems on the a roof on a building is o the area, A, of the ers an area of 36 m <sup>2</sup> . ip between C and A. n it costs to cover a of 24 m <sup>2</sup> . D00,000. What area dig a well is indirectly mber of men, P, es 2 days for 5 men to ip between T and N. it will take 4 men he rate to dig the in the Independent ard.

- 1. **Say**: We have 2 quantities, y and x, which are directly proportional. How do we write this as a ratio?
- 2. Allow pupils to call out their answers. (Answer: y : x)
- 3. Ask: How do we write this as a proportion? Raise your hand. (Answer:  $y \propto x$ )
- 4. Say: What if the quantities are indirectly proportional. What do we write? Raise your hand. (Answer:  $y : \frac{1}{x}$  and  $y \propto \frac{1}{x}$ )
- 5. **Say**: Today we are going to solve direct and indirect proportion problems.

# Introduction to the New Material (10 minutes)

1. Say: Who can explain to the class how to solve the first part of Question a.? Raise your hand.

2. Guide a pupil who has raised their hand to come to the board and solve Question a., part i. (Answer: As shown below)

i.	С	x	А	Cost is directly proportional to area
	С	=	k A	• where k is the constant of proportionality
	18000000	=	$k \times 36$	$\checkmark$ Using C = Le 18,000,000 when A = 36 m <sup>2</sup>
	k	=	$\frac{18000000}{36}$	
			$\frac{500000}{1}$	Divide by a common factor of 36
			Le 500,000/m <sup>2</sup>	Cost per square metre of roof
	С	=	500000A	Equation relating C and A

- 3. Ask pupils to work in pairs to solve parts ii and iii. Give them 3 minutes.
- 4. Ask pupils to raise their hand to answer. Guide pupils to explain how to solve parts ii and iii. on the board.

ii.C=
$$500000A$$
= $500000 \times 24$  $\leftarrow$ Using A = 24 m²=Le 12000000 $\leftarrow$ Cost of roof with area 24 m²iii.C= $500000A$ 15000000= $500000A$  $\leftarrow$ A= $\frac{1500000}{500000}$  $\leftarrow$ A= $\frac{1500000}{500000}$  $\leftarrow$ Divide cost by k to find area $30 m²$  $\leftarrow$ 

- 5. Stop here and check that pupils understand everything up to this point.
- 6. Ask: What did we do in part i of the question to find k? Raise your hand.
- 7. Guide pupils to look at the initial information given. (Answer: We used the information given that the cost of Le 18,000,000 was used to cover an area of 36 m<sup>2</sup>.)
- 8. Ask: What did we do next? Raise your hand.
- 9. Guide pupils to look at the next part where k can be used to find the cost for a given area.
- 10. **Say**: Once we know k, the constant of proportionality, we can find any of the other missing quantities. You can see how that works in part iii. of the question when we have cost and we are asked to find the area.
- 11. **Say**: This is a standard method when solving problems on direct and indirect proportion. The initial information allows us to find k, the constant of proportionality. We then use k to solve for any quantities required. Let us see how it works for a question on indirect proportionality.

### **Guided Practice** (10 minutes)

- 1. Have a pupil from the front of the classroom volunteer to read Question b.
- 2. Ask: What is our first step? Raise your hand.
- 3. Have a pupil from the back of the class answer. (Answer: Use the initial information given to find k. Then find the equation relating time and the number of men.)
- 4. Ask pupils to continue working in pairs to solve Question b. part i.

- 5. Give pupils 2 minutes. Stop to check that all of them have the correct relationship connecting T and N.
- 6. Have a pupil from the front of the classroom volunteer to show on the board how to solve part i. (Answer: As shown below)



- 7. Correct any errors on the calculation on the board. Ask pupils to check their work.
- 8. Say: Now use the equation to answer part ii.
- 9. Allow a further 1 minute. Walk around, if possible, to check the answers and clear up any misconceptions.
- 10. Have a volunteer come to the board to solve part ii.
- 11. Correct any errors on the calculation on the board. Ask pupils to check their work.

(Answer: ii. T = 
$$\frac{10}{N} = \frac{10}{4} = 2.5$$
 days)

### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to solve the problems.
- 2. Point to the following problems on the board:
  - a. The amount of sugar, S, required to make ginger beer is directly proportional to the amount,
     G, of ginger beer being made. It requires 280 g of sugar to make 2 litres of ginger beer.
    - i. Find the constant of proportionality, k.
    - ii. How much sugar is required to make 6 litres of ginger beer?
  - b. The time, t, it takes to plant a field is indirectly proportional to the number of people, p, available. It takes 5 days for 15 people to plant the field.
    - i. Find the constant of proportionality, k.
    - ii. How long will it take 25 people, working at the same rate, to plant the field?
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to some to the board and show their answers to the questions. Ask other pupils to observe to check whether they agree or not.
- 5. Correct any errors on the calculation on the board. Ask pupils to check their work.

(Answers: c. i. 
$$S \propto G$$
,  $S = k G$ ,  $280 = 2k$ ,  $k = \frac{280}{2} = 140 \text{ g/l}$ ; ii.  $S = k G = 140 \times 6 = 840 \text{ g}$   
d. i.  $t \propto \frac{1}{p}$ ,  $t = \frac{k}{p}$ ,  $5 = \frac{k}{15}$ ,  $k = 5 \times 15 = 75$ ; ii.  $t = \frac{75}{p} = \frac{75}{25} = 3 \text{ days}$ )

- 1. Ask: How do we find the constant of proportionality in a standard proportion problem?
- 2. Have a pupil from the back of the classroom volunteer to answer. (Answer: Use the initial information given in the problem.)

Lesson Title: Financial Literacy I	Theme: Everyday Arithmetic	
Lesson Number: M-09-029	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes By the end of the lesson, pupils will be	Teaching Aids None	Preparation 1. Write on the board:	
able to solve problems with		Tax Rate	
wages, salaries, and income		Not over Le 500,000 per month	Nil
tax.		Next Le 500,000 per month	15%
		Next Le 500,000 per month	20%
		Next Le 500,000 per month	30%
		Above Le 2 million per month:	35%
		2. Write the problems from the	
		Guided Practice section on the b	oard.
		3. Write the problems from the	
		Independent Practice section on	the
		board.	

- 1. **Say**: The money that people earn is taxed by the government. It is used to provide services to the country such as education, health and social welfare.
- 2. **Ask**: What is the name of the tax that is paid on the money people earn? Raise your hand. (Answer: Income tax)
- 3. Change the year in the next question from 2015 to the current year. The income tax rates table also changes to the current rates. Use the current income tax rates in the questions.
- 4. Point to the table on the board.
- 5. **Ask**: Who can tell the class, what the tax-free income is? This is the income below which you do not have to pay any income tax? (Answer: Le 500,000 per month)
- 6. Say: Today we are going to solve problems with wages, salaries, and income tax.

# Introduction to the New Material (10 minutes)

- 1. **Say:** Some of you may be wondering what the difference is between a wage and a salary. Wages refer to the money people earn who get paid by the hour or by the day. They get paid at the end of the month only for the hours or days they work. Salaries do not depend on the number of hours or days someone works. It is a fixed amount paid at the end of the month regardless of the number of hours or days someone works.
- 2. **Say:** No matter whether you earn a wage or a salary, you still have to pay income tax on it. We will now do some simple problems on wages and salaries. We will also see how the income tax rates are applied to wages and salaries.
- 3. Write on the board:

Adama earns Le 950,000 per month. She is given a 5% rise in salary. How much is her new monthly salary?

- 4. **Ask:** Who remembers doing questions like this when we were doing percentages? Raise your hand.
- 5. Select a pupil who has raised their hand to carry out the calculations on the board. Guide as needed.

```
100% + 5% = 105% • Add the percentage rise to 100%
```

 $= \frac{105}{100} = 1.05$ Convert to a decimal
New salary = 1.05 × 950000
Le 997,500
Multiply by original income
New salary

- 6. Write on the board: How much income tax does Adama pay per month?
- 7. **Say**: Look at the table of income tax rates on the board.
- 8. Ask: What is the first thing we need to do to find the income tax? Raise your hand.
- 9. Guide a pupil to think about what tax-free income means in practice. (Answer: We need to find the taxable income. We need to subtract tax-free income from new income).
- 10. Ask pupils to use the table and work in pairs for 3 minutes to discuss and share their ideas.
- 11. Say: Please raise your hand when you finish.
- 12. Select one of the last pupils who raised their hand to explain on the board how to solve the problem. Ask other pupils to observe carefully to see if they agree with the calculation.
- 13. Correct any errors in the calculation on the board. Ask pupils to check their work. Example calculation:

Taxable income	= 997,500 - 500.000	•	-Subtract tax-free from new income
	= 497,500	•	—Taxable income
Tax on Le 497,500	= 15% of Le 497,500	•	_Tax rate from table, Adama pays only 15% as her taxable income is less than Le 500,000
	= 0.15 × Le 497.500	•	–Multiply the tax rate and the taxable income
	= Le 74,625	•	–Adama pays Le74,625 in tax

**Guided Practice** (10 minutes)

- 1. Ask pupils to continue to work in pairs.
- 2. Point to the questions on the board:
  - Sallieu earns a daily wage of Le 50,000.
    - a. How much does he earn in 22 days?
    - b. How much tax does he pay on his wages?
- 3. Guide pupils to break down the taxable income to Le 500,000 @ 15% and Le 100,000 @ 20%.
- 4. Walk around, if possible, to check the answers and clear up any misconceptions.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work.

(Answers: a.  $22 \times Le \ 50,000 = Le \ 1,100,000$ ;

b. Taxable income = Le 1,100,000 – 50,0000 = Le 600,000

Tax on Le 600,000 = 15% of Le 500,000 + 20% of Le 100,000

= 0.15 × 500,000 + 0.2 × 100,000 = 75,000 + 20,000 = Le 95,000).

# **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:
  - a. A manager earns Le 4,000,000 per month. Please calculate the income tax paid by the manager per year.
  - b. An office worker earns Le 1,200,000 per month. His salary was increased by 10%. What is his new salary? How much <u>extra</u> income tax does he pay on his new salary?
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.

- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. Taxable income in one month = Le 4,000,000 - 500.000 = 3,500,000, Tax on Le 3,500,000 = 0.15 × 500,000 + 0.2 × 500,000 + 0.3 × 500,000 + 0.35 × 2,000,000 = 75,000 + 100,000 + 150,000 + 700,000 = Le 1,025,000; Income tax per year = 12 × Le1,025,000 = Le 12,300,000 b. New salary = 1.1 × Le 1,200,000 = Le 1,320,000; Additional income = Le 1,320,000 - 1,200,000 = 120,000. From the table, the tax rate for the additional Le 120,000 = 0.2 × 120,000 = Le 24,000.)

- 1. Say: Please write down 2 different things that you learned today.
- 2. Allow pupils one minute to discuss and share their ideas.
- Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: The difference between a wage and salary; how to calculate income tax; how to find a percentage increase)

Lesson Title: Financial Literacy 2Theme: Everyday Arithmetic				
Lesson Number: M-09-030	Class/Level: JSS 3 Time: 35 minutes		Time: 35 minutes	
Learning Outcomes	f Teaching	Aids	<b>Preparation</b>	
By the end of the	None None		1. Write on the bo	ard:
lesson, pupils will be			a. Find 10% of 500	,000.
able to solve simple interest			b. Find the simple interes	st on
problems.			Le 500,000 for one-year	at an interest rate
			of 10% per annum.	
			c. Find the simple interes	st on
			Le 400,000 for 3 years at	an interest rate of
			5% per annum.	
			d. Find the total amount	to repay if a loan of
			Le 200,000 is taken out a	t an interest rate of
			11% for 1 year.	
			2. Write the problems fro	om the Guided
			Practice section on the b	oard.
			3. Write the problems fro	om the
			Independent Practice see	ction on the board.

- 1. Say: Please calculate 10% of 500,000.
- 2. Have a pupil from the back of the classroom to write the answer. Correct any errors. (Answer:  $10\% = \frac{10}{100} = 0.1$ ;  $0.1 \times 500,000 = 50,000$ )
- 3. Say: Today we are going to solve simple interest problems.

# Introduction to the New Material (10 minutes)

- 1. **Say**: When someone puts money in a bank, the bank pays them an additional amount of money. This money depends on the amount put in by the person and how long they keep the money in the bank. Simple interest is calculated as a percentage of the money put in the bank on an annual, or yearly, rate. You do not need to memorise a formula. Just remember how to calculate with percentages.
- 2. **Say**. Please look at Question b. on the board. Who can tell the class, in their own words, what the question is actually asking? Raise your hand.
- 3. Select a pupil who has raised their hand. (Example answer: Find 10% of Le 500,000.)
- 4. **Say**: The example on the board gives the amount paid out if Le 500,000 is kept at a bank at an interest of 10% per annum.
- 5. **Say:** We already calculated the answer for Question b. It is a simple percentage calculation.
- 6. **Say**: If the question requires you to find the simple interest for more than one year, simply multiply your answer by the number of years asked.
- 7. **Ask**: What simple calculation is Question c. asking us to do? Raise your hand.
- 8. Select a pupil who has raised their hand. (Answer: Find 5% of Le 400,000, 3 times.)
- Ask the class to calculate the answer.
   (Answer: 0.05 × 400,000 = Le 20,000; 20,000 × 3 = Le 60,000)
- 10. **Say**: Simple interest calculations also apply with loan repayments. Work with a partner to answer Question d.

- 11. Have a pupil volunteer to explain on the board how to solve Question d. Ask other pupils to observe carefully to see if they agree with the calculation.
- 12. Ask if anyone has used a different method to explain their method on the board.
- 13. Say: This is a percentage increase problem. Le 200,000 has been increased by 11%.
  - (Answer: d.  $0.11 \times 200,000 = 22,000; 200,000 + 22,000 = Le222,000$ ).
- 14. **Say**: If the loan is for more than one-year long, multiply the interest rate by the number of years the loan is for.

## **Guided Practice** (10 minutes)

- 1. Ask pupils to continue to work in pairs.
- 2. Point to the questions on the board:
  - a. Find the simple interest on:
    - i. Le 600,000 for 5 years at an interest rate of 8% per annum.
    - ii. Le 100,000 for 4 years at an interest rate of 4½% per annum.
  - b. Find the total amount to repay if a loan of:
    - i. Le 45,000 is taken out at an interest rate of 9% for 1 year.
    - ii. Le 120,000 is taken out at an interest rate of 7% for 3 years.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. i.  $(0.08 \times 600,000) \times 5 = 48,000 \times 5 = Le 240,000;$ 
  - ii.  $(0.045 \times 100,000) \times 4 = 4,500 \times 4 = \text{Le } 18,000;$
  - b. i. 0.09 × 45,000 = 4,050, 45,000 + 4,050 = Le 49,050;
  - ii.  $(0.07 \times 120,000) \times 3 = 8,400 \times 3 = \text{Le } 25,200; 120,000 + 25,200 = \text{Le } 145,200)$

### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:
  - a. Find the simple interest on:
    - i. Le 750,000 for 3 years at an interest rate of 6% per annum.
    - ii. Le 300,000 for 5 years at an interest rate of 2½% per annum.
  - b. Find the total amount to repay if a loan of:
    - i. Le 165,000 is taken out at an interest rate of 12% for 2 years. How much is repaid each month?
    - ii. Le 250,000 is taken out at an interest rate of 9% for 5 years.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work.

(Answers: a. i.  $(0.06 \times 750,000) \times 3 = 45,000 \times 3 = \text{Le } 135,000;$ 

- ii.  $(0.025 \times 300,000) \times 5 = 7,500 \times 5 = \text{Le } 37,500;$
- b. i.  $(0.12 \times 165,000) \times 2 = 19,800 \times 2 =$ Le 39,600; 165,000 + 39,600 = Le 204,600;
- $204,600 \div 24 = Le 8,525$  per month;
- ii.  $(0.09 \times 250,000) \times 5 = 22,500 \times 5 = \text{Le } 112,500, 250,000 + 112,500 = \text{Le } 362,500).$

- 1. **Ask:** What type of calculation should we think about when we see simple interest problems? Raise your hand.
- 2. Select a pupil who has raised their hand. (Example answers: Percentages, percentage increases, percentages of quantities)
- 3. **Say:** Simple interest problems are percentage problems. If we know how to calculate percentages of quantities and percentage increases, we can solve problems involving simple interest.

Lesson Title: Index Notation and the Laws of Indices	n Title: Index Notation and the Laws of Indices Theme: Numbers and Numeration	
Lesson Number: M-09-031	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	P Teaching Aids	Preparation
	N/A	
By the end of the lesson,	None	1. Write the problems
pupils will be able to:		from the Guided Practice
1. Interpret numbers in index		section on the board.
notation.		2. Write the problems from the
2. State the 6 laws of indices and		Independent Practice section on
solve simple examples for each.		the board.

- 1. Write on the board: 3<sup>4.</sup>
- 2. Say: Please raise your hand if you can expand the number on the board.
- 3. Select a pupil who has raised their hand to write the expansion on the board. (Answer:  $3^4 = 3 \times 3 \times 3 \times 3 = 81$ )
- 4. Ask: How do we read the number on the board? (Answer: Read as "three to the power four".)
- 5. **Say**: In today's lesson, we are going to interpret numbers in index notation. We will also look at the 6 laws of indices and solve simple examples for each.

### Introduction to the New Material (15 minutes)

- 1. Point to the number  $3^4$  on the board.
- 2. **Say**: We call the 3 the base. The 4 is the index. The index tells us how many of the base numbers to multiply together. It is also called the 'power'. Note that the plural of index is 'indices'.
- 3. Write this on the board as shown.
- Say: When we write numbers in this way, we call it 'index notation'. Questions often say to leave our answers in index notation. So, we do not need to calculate what the answer is for 3<sup>4</sup>.
- Say: Let us now look at the rules we use when we do calculations with numbers written with indices.
   We call these rules the laws of indices.
- 6. Write on the board: <u>First law of indices</u> Write on the board:  $a^m \times a^n = a^{m+n}$
- 7. **Ask:** Who can explain to the class what the first law of indices states?
- 8. Guide a pupil to say that: When we multiply expressions (or numbers) with the same base, we add the indices.
- 9. Write on the board:  $2^5 \times 2^3$
- 10. Ask the class to use the law to work out the answer.
- 11. Have a pupil volunteer to solve the question on the board. (Answer: As shown in the table under 'First law of indices'.)
- 12. Write the heading and the second law of indices on the board.
- 13. Guide a pupil to say that: When we divide expressions (or numbers) with the same base, we subtract the indices.

index
<b>3</b> <sup>34</sup>
base
The index (or power) tells us how
many of the base number to
multiply together.
Note: The plural of index is 'indices'

First Law of Indices	Second Law of Indices
$a^m \times a^n = a^{m+n}$	$\frac{a^m}{a^n} = a^m \div a^n = a^{m-n}$
$2^5 \times 2^3 = 2^{5+3}$	$\frac{6^5}{6^3} = 6^5 \div 6^3 = 6^{5 \cdot 3}$
- 25	= 0-
Third Law of Indices	Fourth Law of Indices
$a^0 = 1$	$(a^m)^n = a^{mn}$
$\frac{6^5}{6^3} = 6^5 \div 6^5 = 6^{5\cdot 5}$	$(3^3)^2 = 3^{3^{n_2}}$
= 60	= 36
= 1	
Fifth Law of Indices	Sixth Law of Indices
$(a \times b)^n = a^n \times b^n$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
$(2 \times 3)^4 = 2^4 \times 3^4$	$\left(\frac{3}{5}\right)^3 = \frac{3^3}{5^3}$

- 14. Write on the board:  $\frac{6^5}{6^3}$
- 15. Ask pupils to work in pairs to solve the third law of indices.
- 16. Have pupils volunteer to call out the answer. **Write** it on the board. (Answer: As shown in the table under 'Third law of indices').
- 17. **Say**: Sometimes when we subtract the indices we end up with nothing or zero. The third law of indices explains what to do when this happens.
- 18. Write the information for the third law on the board.
- 19. **Say**:  $6^{0}$  gives 1 because when we divide a number by itself we get 1.
- 20. Point out on the example where we are dividing 6<sup>5</sup> by itself.
- 21. **Say**: In the fourth law, we have an index raised to another index. In this case, we multiply the 2 indices as shown on the board. The fifth and sixth laws deal with bases that are different.
- 22. Write the fifth law and ask pupils to work out the example:  $(2 \times 3)^4$ . (Answer: As shown in the table).
- 23. Write the sixth law and ask pupils to work out the example:  $\left(\frac{3}{5}\right)^3$ . (Answer: As shown in the table).

# **Guided Practice** (5 minutes)

- 1. Ask the pupils to work in pairs to answer the questions on the board.
- 2. Point to the questions on the board:
  - a. Expand:
  - i. 2<sup>3</sup> ii. 2<sup>4</sup>
  - b. Find  $2^3 \times 2^4$
  - i. using expansion ii. using the first law of indices
  - c. Explain your answers to Question b.
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.

# **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions on the board.
- 2. Point to the questions on the board:

Please give the answers to the following expressions in index notation:

a. (5 <sup>2</sup> ) <sup>3</sup>	b.	$7^{5} \div 7^{3}$
c. $4^3 \times 4^4$	d.	(4 × 6) <sup>5</sup>
e. $\frac{8^5}{8^2 \times 8^3}$	f.	$\left(\frac{4}{7}\right)^3$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work.

7. (Answers: a. 
$$(5^2)^3 = 5^{2\times 3} = 5^6$$
; b.  $7^5 \div 7^3 = 7^{5-3} = 7^2$ ; c.  $4^3 \times 4^4 = 4^{3+4} = 4^7$ ; d.  $(4 \times 6)^5 = 4^5 \times 6^5$   
e.  $\frac{8^5}{8^2 \times 3^3} = \frac{8^5}{8^{2+3}} = \frac{8^5}{8^5} = 8^{5-5} = 8^0 = 1$ ; f.  $\left(\frac{4}{7}\right)^3 = \frac{4^3}{7^3}$ ).

### **Closing** (2 minutes)

- 1. **Say:** Look at the sixth law of indices. There is something important in this law we need to be careful about.
- 2. **Ask:** Who can tell me what it is? Raise your hand.
- 3. Select a pupil who has raised their hand to answer.
- 4. If no hand is raised, guide a pupil to say that 'the denominator must not be zero'.
- 5. **Ask**: Why can't we have zero in the denominator? (Answer: We cannot divide by zero; we will not get a real answer)

First Law: multiplication of indices	$a^m \times a^n = a^{m+n}$
Second Law: division of indices	$a^m \div a^n = a^{m-n}$
Third Law: power of zero	$(a^0 = 1)$
Fourth Law: powers of indices	$(a^m)^n = a^{mn}$
Fifth Law: power of a product	$(\mathbf{a} \times \mathbf{b})^{\mathbf{n}} = \mathbf{a}^{\mathbf{n}} \times \mathbf{b}^{\mathbf{n}}$
Sixth Law: power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = b \neq 0$

### [LAWS OF INDICES]

Lesson Title: Application of the Laws of Indices	Theme: Numbers and Numerat	ion
Lesson Number: M-09-032	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	A Teaching Aids	Preparation
By the end of the	None	1. Write the questions from the
lesson, pupils will be		Guided Practice section on the board.
able to apply the 6 laws of		2. Write the questions from the Independent
indices to simplify problems.		Practice section on the board.

- 1. Ask: Who can remind the class of what we were doing in the last lesson? Raise your hand.
- 2. Select a pupil who has raised their hand. (Example answer: Index notation and the 6 laws of indices. Accept other reasonable answers).
- 3. **Say**: Today we are going to apply the laws of indices to simplify problems.

### Introduction to the New Material (10 minutes)

- 1. Say: We can use the laws of indices to simplify expressions with variables.
- 2. Remind the pupils to copy the information on the board throughout the lesson.
- 3. Write on the board: Simplify:  $x^2 \times x^5$
- 4. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

 $x^2 \times x^5 = x^{2+5}$  Use the first law  $a^m \times a^n = a^{m+n}$ =  $x^7$  Add the indices

- 6. Write on the board: Simplify:  $\frac{p^7}{p^4}$
- 7. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 8. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

 $\frac{p^7}{p^4} = p^{7-4}$  Use the second law  $a^m \div a^n = a^{m-n}$ =  $p^3$  Subtract the indices

- 9. Write on the board: Simplify:  $\frac{x^2 \times x^3}{x^5}$
- 10. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 11. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

 $\frac{x^2 \times x^3}{x^5} = \frac{x^{2+3}}{x^5} = x^{(2+3)-5} = x^0 = 1$ 

Use the first, second, and third laws  $(a^0 = 1)$ 

- 12. Write on the board: Simplify:  $(u^3)^2$
- 13. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 14. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

 $\begin{array}{rcl} (u^3)^2 & = & u^{3\times 2} & & \mbox{Use the fourth law } (a^m)^n = a^{mn} \\ & = & u^6 & & \mbox{Multiply the indices} \end{array}$ 

- 15. Write on the board: Simplify:  $(a \times b)^3$
- 16. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 17. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$(a \times b)^3 = a^3 \times b^3$$
 Use the fifth law  $(a \times b)^n = a^n \times b^n$   
=  $a^3 b^3$  Simplify

- 18. Write on the board: Simplify  $\left(\frac{c}{d}\right)^2$
- 19. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 20. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{c}{d}\right)^2 = \frac{c^2}{d^2}$$
 Use the sixth law  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} = b \neq 0$ 

21. Say: We will now apply the laws to more complex expressions.

#### **Guided Practice** (10 minutes)

- 1. Ask the pupils to work in pairs to solve the problems on the laws of indices.
- 2. Point to the questions on the board:

Please give the answers to the following expressions in index notation:

a. 
$$(c^{3})^{2}$$
  
b.  $q^{5} \div q^{3}$   
c.  $z^{3} \times z^{4}$   
d.  $(a^{2} \times b)^{3}$   
e.  $\frac{s^{6}}{s^{2} \times s^{4}}$   
f.  $\left(\frac{u^{2}}{v}\right)^{3}$ 

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a.  $(c^3)^2 = c^{3\times 2} = c^6$ ; b.  $q^5 \div q^3 = q^{5-3} = q^2$ ; c.  $z^3 \times z^4 = z^{3+4} = z^7$ ;

d. 
$$(a^2 \times b)^3 = a^{2 \times 3} \times b^3 = a^6 \times b^3 = a^6 b^3$$
; e.  $\frac{s^6}{s^2 \times s^4} = \frac{s^6}{s^{2+4}} = s^{6-(2+4)} = s^{6-6} = s^0 = 1$   
f.  $\left(\frac{u^2}{v}\right)^3 = \frac{u^{2 \times 3}}{v^3} = \frac{u^6}{v^3}$ .

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions on the board.
- 2. Point to the questions on the board:

Simplify the following expressions giving your answers in index notation:

a. 
$$\frac{4p^2q^4}{2pq^2}$$
  
b. 
$$xy^2 \times x^2y$$
  
c. 
$$(p \times q \times r)^3$$
  
d. 
$$(5r^3s^2)^2$$
  
e. 
$$\left(\frac{a^2b}{ba^2}\right)^0$$
  
f. 
$$\left(\frac{a^4b^2}{a^2b}\right)^2$$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work.

(Answers: a. 
$$\frac{4p^2q^4}{2pq^2} = \frac{4}{2}p^{2-1}q^{4-2} = 2pq^2$$
; b.  $xy^2 \times x^2y = x^{1+2}y^{2+1} = x^3y^3$   
c.  $(p \times q \times r)^3 = p^3 \times q^3 \times r^3 = p^3q^3r^3$ ; d.  $(5r^3s^2)^2 = 25r^{3\times 2}s^{2\times 2} = 25r^6s^4$   
e.  $\left(\frac{a^2b}{ba^2}\right)^0 = 1$ ; f.  $\left(\frac{a^4b^2}{a^2b}\right)^2 = (a^{4-2}b^{2-1})^2 = (a^2b)^2 = a^4b^2$ ).

- 1. Write on the board: Simplify  $\frac{s^3}{s^4}$
- 2. Give pupils 2 minutes to solve the question.
- 3. Ask: What answer do you get? Raise your hand.
- 4. Pupils might be unsure of their answer. Encourage them to raise their hands to share their answers.
- 5. Select a pupil who has raised their hand to explain their answer. (Answer:  $\frac{s^3}{s^4} = s^{3-4} = s^{-1}$ ) 6. Say: We will look at the set in
- 6. Say: We will look at these types of expressions in more detail in our next lesson.

Lesson Title: Indices with Negative Powers	Theme: Numbers and Numerat	ion
Lesson Number: M-09-033	Class/Level: JSS 3	Time: 35 minutes

<ul> <li>Learning Outcomes         By the end of the             lesson, pupils will be             able to:      </li> <li>Identify that a number with a             negative index can be rewritten             as a fraction (a<sup>-n</sup> = 1/a<sup>n</sup>).         </li> <li>Apply the laws for multiplying             and dividing indices to those         </li> </ul>	None	Preparation 1. Write on the board: Simplify $\frac{a^4}{a^6}$ a. using the laws of indices b. using expansion 2. Write the questions from the Guided Practice section on the board. 2. Write the questions from the
2. Apply the laws for multiplying and dividing indices to those with positive and negative		Practice section on the board. 3. Write the questions from the Independent Practice section on the board.
powers.		

- 1. Write on the board: Simplify  $\frac{a^4}{a^5}$
- 2. Say: Please raise your hand when you have the answer.
- 3. Select a pupil who has raised their hand to explain their answer. (Answer:  $\frac{a^4}{a^5} = a^{4-5} = a^{-1}$ )
- 4. **Say**: Today we are going to identify that a number with a negative index can be rewritten as a fraction. We will apply the laws to positive and negative powers.

## Introduction to the New Material (10 minutes)

- 1. Ask: Who can explain to the class how to expand  $\frac{a^4}{a^5}$ ? Raise your hand.
- 2. Select a pupil who has raised their hand to show the expansion on the board. Ask the others to solve the question in their exercise books.
- 3. Correct any errors in the calculation on the board. Ask pupils to check their work.

(Answer:  $\frac{a^4}{a^5} = \frac{a \times a \times a \times a}{a \times a \times a \times a \times a} = \frac{1}{a}$ )

- 4. Ask: Can anyone tell the class what they notice about the 2 answers? Raise your hand.
- 5. Select a pupil who has raised their hand to make the connection between the 2 answers.

(Answer: They are the same,  $a^{-1} = \frac{1}{a}$ ).

- 6. **Say**: Answer the question on the board:  $\frac{a^4}{a^6}$ . You have 1 minute to think and write down your ideas.
- 7. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
- 8. Ask: Who would like to share their ideas with the class? Raise your hand.
- 9. Select a pupil who has raised their hand to show the solution for part a. on the board. Select another pupil to show part b.

(Answer: i. 
$$\frac{a^4}{a^6} = a^{4-6} = a^{-2}$$
; ii.  $\frac{a^4}{a^6} = \frac{a \times a \times a \times a}{a \times a \times a \times a \times a \times a} = \frac{1}{a \times a} = \frac{1}{a^2}$ ).

- 10. Correct any errors in the calculation on the board. Ask pupils to check their work.
- 11. Write on the board:

$$a^{-2} = \frac{1}{a^2} = \frac{1}{a \times a}$$
$$a^{-3} = \frac{1}{a^3} = \frac{1}{a \times a \times a}$$

12. **Say**: In general:  $a^{-n} = \frac{1}{a^n}$ , where n is a positive integer.

- 13. Ask the pupils to copy the information on the board. Remind them to take notes throughout the lesson.
- 14. Say: All the laws of indices apply to negative indices as well.
- 15. Show on the board how the expression  $4^2 \times 4^{-3}$  is simplified.

$$4^2 \times 4^{-3} = 4^{2+(-3)} = 4^{-1} = \frac{1}{4}$$

- 16. Have a pupil volunteer to come to the board to simplify  $\frac{5^7}{5^9}$ . Ask other pupils to solve it in their exercise books.
- 17. Correct any errors in the calculation on the board. Ask pupils to check their work.

$$\frac{5^{7}}{5^{9}} = 5^{7-9} = 5^{7-9} = 5^{-2} = \frac{1}{5^{2}} = \frac{1}{5\times 5} = \frac{1}{25}$$

18. Write:  $(2^3)^{-2}$ 

- 19. Ask pupils to work in pairs to simplify  $(2^3)^{-2}$
- 20. Have a pupil volunteer to explain how to simplify the expression on the board. Ask other pupils to solve it in their exercise books.
- 21. Correct any errors in the calculation on the board. Ask pupils to check their work.

$$(2^{3})^{-2} = 2^{-6}$$
  
=  $\frac{1}{2^{6}}$   
=  $\frac{1}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{64}$ 

- 22. Explain to pupils that it is not usually necessary to find the value of the expression. It can be left in index notation.
- 23. Say: We can also use negative indices with variables.

### **Guided Practice** (10 minutes)

- 1. Ask the pupils to continue to work in pairs.
- 2. Say: Apply the laws of indices so the expressions do not contain any negative indices.
- 3. Point to the questions on the board:

Please give the answers to the following expressions in index notation:

g. 
$$(c^{-3})^2$$
  
i.  $z^3 \times z^{-4}$   
j.  $(a \times b)^{-2}$   
k.  $\left(\frac{1}{w}\right)^{-3}$   
l.  $\left(\frac{u^2}{v}\right)^{-3}$ 

- 4. Walk around, if possible, to check the answers and clear up any misconceptions.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. Pay particular attention to questions e. and f.

(Answers: a. 
$$(c^{-3})^2 = c^{-3\times 2} = c^{-6} = \left(\frac{1}{c^6}\right)$$
; b.  $q^3 \div q^5 = q^{3-5} = q^{-2} = \left(\frac{1}{q^2}\right)$ ;  
c.  $z^3 \times z^{-4} = z^{3+(-4)} = z^{-1} = \left(\frac{1}{z}\right)$ ; d.  $(a \times b)^{-2} = a^{-2} \times b^{-2} = a^{-2}b^{-2} = \left(\frac{1}{a^2b^2}\right)$ ;  
e.  $\left(\frac{1}{w}\right)^{-3} = \frac{1}{w^{-3}} = w^3$ ; f.  $\left(\frac{u^2}{v}\right)^{-3} = \frac{u^{2\times(-3)}}{v^{-3}} = \frac{u^{-6}}{v^{-3}} = \frac{u^3}{v^6}$ .

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:

Apply the laws of indices so the expressions do not contain any negative indices:

g. 
$$\frac{p^2q^4}{p^3q^2}$$
  
i.  $(p \times q \times r)^{-3}$   
k.  $\left(\frac{u^2}{v^2}\right)^{-4}$ 

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a.

$$\frac{p^2 q^4}{p^3 q^2} = p^{2-3} q^{4-2} = p^{-1} q^2 = \frac{q^2}{p}; \text{ b. } x^{-2} y^2 \times x^2 y^{-2} = x^{-2+2} y^{2+(-2)} = x^0 y^0 = 1$$
  
c.  $(p \times q \times r)^{-3} = (pqr)^{-3} = \frac{1}{(pqr)^3} = \frac{1}{p^3 q^3 r^3};$   
d.  $(5r^3s^2)^{-2} = \frac{1}{(5r^3s^2)^2} = \frac{1}{25r^{3\times 2}s^{2\times 2}} = \frac{1}{25r^{6}s^4}; \text{ e. } \left(\frac{u^2}{v}\right)^{-4} = \frac{u^{-8}}{v^{-4}} = \frac{v^4}{u^8}$ 

- 1. Ask: Who can tell the class what they learned today? Raise your hand.
- 2. Select a pupil who raised their hand to answer. (Example answer: Learnt about negative indices; how to apply the laws of indices to negative powers)
- 3. **Say:** Index notation is very useful when simplifying expressions and equations in algebra. We will look at fractional indices in the next lesson.

Lesson Title: Indices with Fracti	onal Powers	Theme: Numbers and Numeration
Lesson Number: M-09-034		Class/Level: JSS 3 Time: 35 minutes
Learning Outcomes By the end of the lesson, pupils will be able to: 1. Identify that a number with a fractional power can be rewritten as a root $\left(a^{\frac{1}{n}} = \sqrt[n]{a}\right)$ . 2. Simplify simple indices with	Teaching Aid None	<b>Preparation</b> 1. Write in the centre of the board: Simplify with no fractional powers: $\left(\frac{xyz^2}{x^{2y^3z}}\right)^{-2}$ 2. Draw this table on one side of the board: $\boxed{a^{\frac{1}{2}} = \sqrt{a}  9^{\frac{1}{2}} = 3}{a^{\frac{1}{3}} = \sqrt[3]{a}  8^{\frac{1}{3}} = 2}$
fractional powers.		$a^{\frac{1}{4}} = \sqrt[4]{a} 256^{\frac{1}{4}} = 4$ $a^{\frac{1}{n}} = \sqrt[n]{a}$ 3. Write the questions from the Guided Practice section on the board. 4. Write the questions from the Independent Practice section on the board.

1. **Say**: Simplify the expression on the board:  $\left(\frac{xyz^2}{x^2y^3z}\right)^{-2}$ 

- 3. Have a pupil from the back of the classroom give their answer. (Answer:  $\frac{x^2y^4}{z^2}$ )
- 4. **Say**: Today we are going to identify that a number with a fractional power can be rewritten as a root. We will also solve simple indices with fractional powers.

# Introduction to the New Material (10 minutes)

- 1. **Say**: We sometimes have indices that are fractions. They are used to represent square roots, cube roots and other roots of numbers.
- 2. Point to the table written on the board.
- 3. Say: Take a look at how square roots, cube roots and other roots are represented with indices.
- 4. Point to each line in turn.
- 5. **Say:** a raised to the power of half is the square root of a.
- 6. **Say:** a raised to the power of one-third is the cube root of a.
- 7. Say: The general rule is a raised to the power of one over n and it is the n<sup>th</sup> root of a.
- 8. Ask the pupils to copy the information on the board.
- 9. Say: Let us look at one more example of a square and a cube root.
- 10. Write on the board:

$$25^{-\frac{1}{2}} = \frac{1}{\sqrt{25}} = \frac{1}{5}$$
$$27^{\frac{1}{3}} = \sqrt[3]{27} =$$

- 11. Ask: What is the cube root of 27?
- 12. Allow a few moments for pupils to think.

- 13. Have a pupil from the left-hand side of the classroom volunteer to answer. (Answer: 3)
- 14. Say: Let us simplify some simple numbers with fractional indices.
- 15. Show on the board how to simplify  $(4^2 \times 4^4)^{\frac{1}{2}}$

$$(4^{2} \times 4^{4})^{\frac{1}{2}} = (4^{2+4})^{\frac{1}{2}}$$
$$= (4^{6})^{\frac{1}{2}}$$
$$= 4^{6 \times \frac{1}{2}}$$
$$= 4^{3}$$

- 16. **Say**: This is exactly the same method we used for positive and negative indices. We just need to apply the law and do the calculation.
- 17. Write on the board: Simplify:  $\left(\frac{3^4}{3^2}\right)^{\frac{1}{2}}$
- 18. Allow pupils to think and write down ideas for 1 minute.
- 19. After 1 minute, ask them to pair up with their neighbour and discuss how to solve the problem for another minute.
- 20. Ask: Who would like to share their ideas with the class?
- 21. Select a pupil who has raised their hand to explain on the board how to solve the problem.
- 22. Correct any errors in the calculation on the board. Ask pupils to check their work.

(Answer: 
$$\left(\frac{3^4}{3^2}\right)^{\frac{1}{2}} = (3^{4-2})^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3^{2\times\frac{1}{2}} = 3^1 = 3$$
)

### **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Point to the questions on the board: Calculate the following roots:

m. 
$$49^{\frac{1}{2}}$$
  
n.  $49^{-\frac{1}{2}}$   
p.  $32^{-\frac{1}{5}}$   
q.  $(5^3 \times 5^5)^{\frac{1}{2}}$   
r.  $\left(\frac{9 \times 27}{3}\right)^{\frac{1}{4}}$ 

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions on the board.
- 5. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: a.  $49^{\frac{1}{2}} = \sqrt{49} = 7$ : h  $49^{-\frac{1}{2}} = \frac{1}{2} = \frac{1}{2}$

$$d. \ 32^{-\frac{1}{5}} = \frac{1}{\frac{5}{\sqrt{32}}} = \frac{1}{2}; \ e. \ (5^3 \times 5^5)^{\frac{1}{2}} = \ (5^{3+5})^{\frac{1}{2}} = (5^8)^{\frac{1}{2}} = 5^{8\times\frac{1}{2}} = 5^4;$$
$$f. \left(\frac{9\times27}{3}\right)^{\frac{1}{4}} = (3\times27)^{\frac{1}{4}} = (3\times3^3)^{\frac{1}{4}} = (3^{1+3})^{\frac{1}{4}} = (3^4)^{\frac{1}{4}} = 3^{4\times\frac{1}{4}} = 3).$$

#### Independent Practice (10 minutes)

- 1. Ask the pupils to continue to work in pairs to answer the questions.
- 2. Tell each pupil to take one question to answer. After they have solved their question, pupil 1 should explain how they worked out the answer to their question to pupil 2. Pupil 2 will check

that their method and answer are correct. Then pupil 2 will share the solution for their question and pupil 1 will check the method and answer.

3. Point to the questions on the board:

Please calculate the following roots:

a. 
$$4^{-\frac{1}{2}} + 4^{\frac{1}{2}}$$
 b.  $8^{\frac{1}{3}} - 8^{-\frac{1}{3}}$ 

- 4. Walk around, if possible, to check the answers and clear up any misconceptions.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a.

$$4^{-\frac{1}{2}} + 4^{\frac{1}{2}} = \frac{1}{\sqrt{4}} + \sqrt{4} = \frac{1}{2} + 2 = 2\frac{1}{2}$$
  
b.  $8^{\frac{1}{3}} - 8^{-\frac{1}{3}} = \sqrt[3]{8} - \frac{1}{\sqrt{8}} = 2 - \frac{1}{2} = 1\frac{1}{2}$ 

- 1. Say: Please write your name on a piece of paper. Solve the following problem on your paper.
- 2. Write on the board: Simplify  $(3^4 \times 3^6)^{\frac{1}{2}}$
- 3. Collect the papers at the end of the lesson.
- 4. Check the pupils' work to get an idea of their level of understanding. (Answer:  $(3^4 \times 3^6)^{\frac{1}{2}} = (3^{4+6})^{\frac{1}{2}} = (3^{10})^{\frac{1}{2}} = 3^{10 \times \frac{1}{2}} = 3^5$ )
- 5. Address any issues in the next lesson when pupils will be practicing more problems on fractional indices.

Lesson Title: Multiplying and Dividing Indices	Theme: Numbers and Numerat	ion
with Fractional Powers		
Lesson Number: M-09-035	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None None	1. Write in the centre of the
lesson, pupils will be		board:
able to apply the laws for		Simplify with no fractional powers:
multiplying and dividing indices to those with positive and negative fractional powers.		$\left(\frac{2^6}{2^2}\right)^{\frac{1}{2}}$ 2. Write the questions from the Guided Practice section on the board. 3. Write the questions from the Independent Practice section on the board.

- 1. Say: In the last lesson, we were looking at expressions which have fractional powers.
- 2. **Say**: Please simplify the expression on the board:  $\left(\frac{2^6}{2^2}\right)^{\frac{1}{2}}$
- 3. After 1 minute ask pupils to raise their hand to share the answer. Select a pupil from the back of the classroom to give the answer to the problem.

(Answer:  $\left(\frac{2^6}{2^2}\right)^{\frac{1}{2}} = (2^{6-2})^{\frac{1}{2}} = (2^4)^{\frac{1}{2}} = 2^{4 \times \frac{1}{2}} = 2^2$ )

4. **Say**: Today we are going to apply the laws for multiplying and dividing indices to those with positive and negative fractional powers.

### Introduction to the New Material (10 minutes)

- 1. Remind the pupils to copy the information on the board throughout the lesson.
- 2. Write on the board: Simplify:  $(x^3 \times x^6)^{\frac{1}{3}}$
- 3. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$(x^{3} \times x^{6})^{\frac{1}{3}} = (x^{3+6})^{\frac{1}{3}}$$
$$= x^{9 \times \frac{1}{3}}$$
$$= x^{3}$$

- 5. Write on the board: Simplify:  $\left(\frac{p^8}{p^4}\right)^{\overline{2}}$
- 6. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 7. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{p^8}{p^4}\right)^{\frac{1}{2}} = (p^{8-4})^{\frac{1}{2}}$$

$$= (p^4)^{\frac{1}{2}}$$

$$= p^{4 \times \frac{1}{2}}$$

$$= p^2$$

- 8. Write on the board: Simplify:  $(u^{10})^{-\frac{1}{2}}$
- 9. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 10. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$(u^{10})^{-\frac{1}{2}} = u^{10 \times (-\frac{1}{2})}$$
  
=  $u^{-5}$   
 $(u^{10})^{-\frac{1}{2}} = \frac{1}{u^5}$   
in the board: Simplify:  $\left(\frac{c}{-}\right)^{-\frac{1}{3}}$ 

- 11. Write on the board: Simplify:  $\left(\frac{a}{d}\right)$
- 12. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 13. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{c}{d}\right)^{-\frac{1}{3}} = \left(\frac{d}{c}\right)^{\frac{1}{3}}$$
$$= \frac{d^{\frac{1}{3}}}{c^{\frac{1}{3}}}$$

- 14. Write on the board: Simplify:  $(a \times b)^{\frac{1}{4}}$
- 15. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 16. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$(a \times b)^{\frac{1}{4}} = a^{\frac{1}{4}} \times b^{\frac{1}{4}} = a^{\frac{1}{4}} b^{\frac{1}{4}}$$

- 17. Say: We can apply the laws to even more complex expressions.
- 18. Write on the board: Simplify:  $\left(\frac{a^2b^4}{c^6}\right)^{-\frac{1}{2}}$
- 19. Have a pupil volunteer to come to the board to solve the problem. Ask other pupils to solve it in their exercise books.
- 20. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answer: see below)

$$\left(\frac{a^2b^4}{c^6}\right)^{-\frac{1}{2}} = \left(\frac{c^6}{a^2b^4}\right)^{\frac{1}{2}}$$

$$= \frac{c^{6\times\frac{1}{2}}}{a^{2\times\frac{1}{2}}b^{4\times\frac{1}{2}}}$$
$$= \frac{c^{3}}{ab^{2}}$$

#### **Guided Practice** (10 minutes)

- 1. Ask the pupils to work in pairs.
- 2. Point to the questions on the board:

Please simplify the following expressions:

s. 
$$(x^5 \times x^7)^{\frac{1}{3}}$$
  
u.  $\frac{a^{\frac{1}{2}}}{a}$   
w.  $(ab)^{-\frac{1}{4}}$   
t.  $\frac{a}{a^{\frac{1}{2}}}$   
v.  $(t^9)^{-\frac{1}{3}}$   
x.  $(\frac{u}{v})^{-\frac{1}{4}}$ 

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a.  $(x^5 \times x^7)^{\frac{1}{3}} = (x^{5+7})^{\frac{1}{3}} = (x^{12})^{\frac{1}{3}} = x^{12 \times \frac{1}{3}} = x^4$ ;

b. 
$$\frac{a}{a^{\frac{1}{2}}} = a^{1-\frac{1}{2}} = a^{\frac{1}{2}}$$
; c.  $\frac{a^{\frac{1}{2}}}{a} = a^{\frac{1}{2}-1} = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}}$ ; d.  $(t^9)^{-\frac{1}{3}} = t^{9\times(-\frac{1}{3})} = t^{-3} = \frac{1}{t^3}$ ;  
e.  $(ab)^{-\frac{1}{4}} = \frac{1}{(ab)^{\frac{1}{4}}} = \frac{1}{a^{\frac{1}{4}b^{\frac{1}{4}}}}$ ; f.  $(\frac{u}{v})^{-\frac{1}{4}} = (\frac{v}{u})^{\frac{1}{4}} = \frac{v^{\frac{1}{4}}}{u^{\frac{1}{4}}}$ ).

### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:

Simplify the following expressions:

a. 
$$\frac{p^{\frac{3}{2}}}{p}$$
  
b.  $\frac{p}{p^{\frac{3}{2}}}$   
c.  $(t^{12})^{-\frac{1}{4}}$   
d.  $\frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}}$   
e.  $\left(\frac{1}{y}\right)^{-\frac{1}{5}}$   
f.  $\left(\frac{a^{6}b^{3}}{c^{9}}\right)^{-\frac{1}{3}}$ 

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange their exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a.

$$\frac{p^{\overline{2}}}{p} = p^{\frac{3}{2}-1} = p^{\frac{1}{2}}; \text{ b. } \frac{p}{p^{\frac{3}{2}}} = p^{1-\frac{3}{2}} = p^{-\frac{1}{2}} = \frac{1}{p^{\frac{1}{2}}};$$

c. 
$$(t^{12})^{-\frac{1}{4}} = t^{12 \times (-\frac{1}{4})} = t^{-3} = \frac{1}{t^3}; \text{ d.} \frac{x^{\frac{1}{3}}}{x^{\frac{1}{2}}} = x^{\frac{1}{3} - \frac{1}{2}} = x^{-\frac{1}{6}} = \frac{1}{x^{\frac{1}{6}}}; \text{ e.} \left(\frac{1}{y}\right)^{-\frac{1}{5}} = y^{\frac{1}{5}};$$
  
f.  $\left(\frac{a^6b^3}{c^9}\right)^{-\frac{1}{3}} = \left(\frac{c^9}{a^6b^3}\right)^{\frac{1}{3}} = \frac{c^{9 \times \frac{1}{3}}}{a^{6 \times \frac{1}{3}}b^{3 \times \frac{1}{3}}} = \frac{c^3}{a^2b}$ 

Closing (2 minutes)

- 1. Say: Discuss in your pairs some of the things you learned about index notation.
- 2. Allow one minute for discussion.
- 3. Say: Raise your hand to tell the class one thing you learned.
- 4. Select as many pupils in one minute to say what they learned about index notation.
- 5. Tell pupils if they did not get a chance to speak now, they will be asked to speak on another topic in the future.

(Example answers: Learned about index and base; how to expand numbers written in powers; laws of indices; applying laws to positive, negative, fractional powers)

Lesson Title: Multiplying and Dividing by Powers of 10	Theme: Numbers and Nu	imeration
Lesson Number: M-09-036	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Faching Aids	Preparation
By the end of the	None None	1. Write this set on the board:
lesson, pupils will be		{ 1, 10, 100, 1000, }
able to multiply and divide		2. Write the questions from the
whole numbers and decimals by		Guided Practice section on the board.
powers of 10.		3. Write the questions from the
		Independent Practice section on the
		board.

- 1. **Say**: Please raise your hand if you can tell the next number in the set on the board.
- 2. Select a pupil who has raised their hand to answer. (Answer: 10000)
- 3. Say: Today we are going to multiply and divide whole numbers and decimals by powers of 10.

### Introduction to the New Material (10 minutes)

- 1. **Say**: Think back to our lessons on index notation. Who can write 1 as a power of 10? Raise your hand to answer. (Answer: 10<sup>0</sup>)
- 2. Say: Write a new set showing all the numbers as powers of 10.
- Have a pupil volunteer to come to the board and write the set in powers of 10. (Answer: { 10<sup>0</sup>, 10<sup>1</sup>, 10<sup>2</sup>, 10<sup>3</sup>, ... })
- 4. **Ask:** What do you notice about the indices in the powers of 10? Raise your hand to answer. (Answer: The number of zeros in the number and the index are the same.)
- 5. **Ask**: How would you multiply and divide by powers of 10. Raise your hand to answer. (Answer: To multiply, move the decimal point to the right by the index (or power) of 10. To divide, move the decimal point to the left by the power of 10.)
- 6. Explain to the pupils that if the number is a whole number, then a decimal point is placed at the end before moving the correct number of places.
- 7. Emphasise the relationship between the power of 10 and the number of places to move the decimal. For example, when the power of 10 is 2 for 100, the decimal point is moved 2 places.
- 8. Write on the board:  $58 \times 100$ .
- 9. **Ask**: Where is the decimal point in 58? Raise your hand. (Answer: At the end of the number, after the 8.)
- 10. Ask: What is the power of 10 in 100? Raise your hand to answer. (Answer: 2)
- 11. **Ask**: How many places should the decimal point be moved? Raise your hand to answer. (Answer: 2)
- 12. **Say:** To multiply by 100, move the decimal point 2 places to the right. To do this we will add 2 zeros to the number.
- 13. Show the following explanation on the board:

 $58 \times 100 = 58 \times 10^{2}$ = 58.00 \times 10^{2} = 5800

14. Write on the board:

a.  $457 \times 100$  b.  $3.42 \times 10$  c.  $978 \div 10$  d.  $6.7 \div 1000$ 

- 15. Have pupils from around the classroom volunteer to explain their answers to the questions.
- 16. Ask each pupil to give the power of 10 (underlined) and therefore how many places to move the decimal point. (Answer: a. <u>2</u>, 45700; b. <u>1</u> 34.2; c. <u>1</u>, 97.8; d. <u>3</u>, 6700).
- 17. **Say:** We are now going to start thinking of numbers in a slightly different way to what we have been used to. We often have to multiply and divide with numbers such as 20, 30, 300 and other multiples of powers of 10. It is very useful to know a different way to do this sort of calculation. This helps when we have to calculate with very large and very small numbers.
- 18. Write on the board: a.  $1.32 \times 20$  b.  $24.6 \div 20$
- 19. Explain each calculation as shown below.

a.  $1.32 \times 20 = 1.32 \times 2 \times 10$   $= 2.64 \times 10$  = 26.4b.  $24.6 \div 20 = \frac{24.6}{20}$   $= \frac{24.6}{2 \times 10}$   $= \frac{12.3}{10}$ 1.23

- 20. **Say**: We will start learning how to calculate with very large and very small numbers in the next lesson.
- 21. For today, we will practise calculating with numbers less than 1000.

### **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Point to the questions on the board:

Give the power of 10 for each of the calculations below. Calculate:

a.	7.12  imes 10	b.	2.8  imes 100	с.	23.2 ÷ 100
d.	84.5 ÷ 10	e.	$3.4 \div 1000$	f.	36 ÷ 100
g.	2.9 × 20	h.	52 × 200	i.	3.63 ÷ 30

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers on the board. Ask pupils to check their work. (Answers: The powers of 10 are underlined a. <u>1</u>, 71.2; b. <u>2</u>, 280; c. <u>2</u>, 0.232;
  d. <u>1</u>, 8.45; e. <u>3</u>, 0.0034; f. <u>2</u>, 0.36; g. <u>1</u> 58; h. <u>2</u>, 10,400; i. <u>1</u>, 0.12 )

### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:

Give the power of 10 for each of the calculations below. Calculate:

a.	9.34 × 10	b.	289  imes 1000	с.	0.042 ÷ 10
d.	1.5  imes 9000	e.	45.3 ÷ 30	f.	42 ÷ 3000
g.	25 × 700	h.	615 ÷ 500	i.	$4 \times 6$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- Write the correct answers on the board. Ask pupils to check their work. (Answers: The powers of 10 are underlined. a. <u>1</u>, 93.4; b. <u>3</u>, 289,000;
   c.<u>1</u>, 0.0042; d. <u>3</u>, 13,500; e. <u>1</u>, 1.51; f. <u>3</u>, 0.014; g. 17,500; h. <u>2</u>, 1.23; i. <u>0</u>, 24)

- 1. **Ask:** Why is it useful to learn how to multiply and divide with powers of 10? Raise your hand. (Answer: To help with calculating very large and very small numbers).
- 2. **Say:** Tomorrow we will learn about 'standard form' which is a way of writing very large numbers using powers of 10.

Lesson Title: Standard Form of Large Numbers	Theme: Numbers and Numeration	
Lesson Number: M-09-037	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>	
By the end of the	None	1. Write on the board:	
lesson, pupils will be		Write the power of 10 and calculate:	
able to interpret and write large		a. 5 × 1000 b. 5.25 × 100	
numbers in standard form		c. 4 × 10000 d. 5.37 × 10	
(scientific notation): a $ imes$ 10 $^{n}$		2. Write the questions from the Guided	
where $1 \le a < 10$ and n is an		Practice section on the board.	
integer.		3. Write the questions from the	
		Independent Practice section on the board.	

1. Say: Please look at the questions on the board. You have 1 minute to write the answers:

Write the power of 10 and calculate:

- 2. Have pupils from around the classroom volunteer to give their answers. (Answers: The powers of 10 are underlined. a. <u>3</u>, 5000; b. <u>2</u>, 525; c. <u>4</u>, 40000; d. <u>1</u>, 53.7)
- 3. **Say**: Today we are going to interpret and write large numbers in standard form, also called scientific notation.

### Introduction to the New Material (10 minutes)

- 1. Say: Think back to our lessons on index notation. Write the questions on the board as powers of 10. For example, 5  $\times$  1000 will become 5  $\times$  10<sup>3</sup>
- 2. Allow 1 minute for the pupils to answer.
- 3. Have pupils from around the classroom volunteer to answer. (Answers: ii. 5.25  $\times$  10  $^2$ ; iii. 4  $\times$  10  $^4$ ; iv. 5.37  $\times$  10  $^1$ )
- 4. Write on the board:

 $5 \times 10^{3}$   $5.25 \times 10^{2}$   $4 \times 10^{4}$  $5.37 \times 10^{1}$ 

- 5. **Say**: There are 2 main points we need to recognise in the way we write the numbers. The first one is that the numbers we are multiplying or dividing are all between 1 and 10. Let us check if that is true.
- 6. Guide a pupil to read the numbers. (Answer: 5, 5.25, 4, 5.37)
- 7. **Ask:** Is there any number which does not follow our rule? Raise your hand if you say 'yes'. (Answer: No)
- 8. **Say:** The second thing to recognise is that all the powers of 10 are integers. Let us check if that is true.
- 9. Guide another pupil to read the powers. (Answer: 3, 2, 4, 1)

- 10. **Ask:** Is there any power which does not follow our rule? Raise your hand if you say 'yes' (Answer: No)
- 11. Say: This method of writing a number is called 'standard form.'
- 12. Write on the board:

 $\underline{a \times 10^{n}}$  where  $1 \le a < 10$  and n is an integer

- 13. **Say:** We read this as "a times 10 to the power n" where 'a' is greater than or equal to 1 and less than 10; 'a' is also an integer'. This simply means that 'a' can be a whole number or a decimal number, but 'n' has to be an integer; no fractions or decimals. If 'a' is a fraction, we will change it to a decimal.
- 14. Say: We write numbers in standard form when we want a short way of writing very large or very small numbers. For example, we can write Le 25, 000 as Le  $2.5 \times 10^{4}$ . This is very useful, especially for very large sums of money.
- 15. **Say:** Standard form is also called scientific notation. Science deals with very large numbers such as the distance between the earth and the sun. Science also deals with very small numbers such as the distance between atoms in a molecule.
- 16. **Say:** Today, we are looking at very large numbers. We call any number greater than 0 a large number and we can write it in standard form. We will look at very small numbers in the next lesson. These will be numbers smaller than zero.
- 17. Show on the board how to change another number to standard form.
- 18. Say: Let us change 45,100 to standard form.

$$\begin{array}{rcl} 45100 & = & 4.51 \times 10000 \\ & = & 4.51 \times 10^{-4} \end{array}$$

- 19. **Ask**: Is 'a' equal to or greater than 1, and also less than 10? Raise your hand. (Answer: Yes, it is 4.51.)
- 20. Ask: Is the power of 10 an integer? Raise your hand. (Answer: Yes, it is 4.)
- 21. **Say**: Then we have written the number in standard form.
- 22. **Ask**: If we change back from standard form to the ordinary number, do we get the number we started with? Raise your hand. (Answer: Yes, then we have converted the number correctly to standard form.)
- 23. **Say:** If we have a number in standard form, we already know how to write it as an ordinary number.
- 24. Show the following explanation on the board.

### **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Point to the questions on the board:

Mark with a  $\checkmark$  if the number is in standard form.

Mark with a **×** if the number is not in standard form. Write the number in standard form.

a. $31.7 \times 10^{2}$	b. 2.834 × 10
c. 1875	d. 4 × 10
e. 243 × 10 <sup>-2</sup>	f. 987

Please change these numbers from standard form to ordinary numbers.

- g.  $1.25 \times 10^{6}$  h.  $1.89 \times 10^{4}$
- i.  $8.5 \times 10^{7}$  j.  $3.927 \times 10^{2}$
- 3. Walk around, if possible, to check the answers and clear up any misconceptions. For example, some pupils may think that Question d. is not in standard form. Ask them the questions we used to check above.
  - ✓ Ask: Is 4 equal to or greater than 1, and also less than 10. (Answer: Yes)
  - ✓ **Ask**: Is the power of 10 an integer? (Answer: Yes, the integer is 1)
  - $\checkmark$  Then the number is in standard form.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. ×, 3.17 × 10<sup>3</sup>; b. ✓; c. ×, 1.875 × 10<sup>3</sup>; d ✓; e. ×, 2.43 × 10<sup>4</sup>; f. ×, 9.87 × 10<sup>2</sup>; g. 1,250,000; h. 18,900; i. 85,000,000; j. 392.7)

# **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:

Change these numbers to standard form.

a.	$872.7  imes 10^{-2}$	b.	$2846  imes 10^{-3}$
c.	318,972	d.	$4387 \times 10$
e.	$27\times10$ $^4$	f.	2

Change these numbers from standard form to ordinary numbers.

g.	$7.25 imes10$ $^6$	h.	$9 imes10$ $^4$
i.	$2.173  imes 10^{-7}$	j.	$3.0001 \times 10^{-2}$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. 8.727 × 10<sup>4</sup>; b. 2.846 × 10<sup>6</sup>; c. 3.18972 × 10<sup>5</sup>; d. 4.387 × 10<sup>4</sup>; e. 2.7 × 10<sup>5</sup>; f. 2 × 10<sup>°</sup> (or 2 × 1 or 2); g. 7,250,000; h. 90,000; i. 21,730,000; j. 300.01)

- 1. Ask: What 2 facts show that a number is in standard form? Raise your hand.
- 2. Have a pupil from the back of the classroom volunteer to answer. (Answer: The number must be greater than or equal to 1 and also less than 10, the power of 10 must be an integer.)
- 3. Say: In the next lesson, we will look at the standard form of small numbers. Great work class!
| Lesson Title: Standard Form of Small Numbers | Theme: Numbers and Num | eration          |
|--|------------------------|------------------|
| Lesson Number: M-09-038                      | Class/Level: JSS 3     | Time: 35 minutes |

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None	1. Write on the board:
lesson, pupils will be	<u>M</u> AS	Write the power of 10, then calculate:
able to interpret and write		a. 5 x 100000
small numbers in standard		b. 5 ÷ 1000
form (scientific notation): a $ imes$		2. Write the questions from the Guided
10 <sup>n</sup> where $1 \le a < 10$ and n is		Practice section on the board.
an integer.		3. Write the questions from the Independent
-		Practice section on the board.

- 1. Ask: Who can remind the class of what we did in the last lesson? Raise your hand.
- 2. Select a pupil who has raised their hand to answer. (Example answers: We learned about standard form; how to write numbers with multiples of 10; using index notation to write numbers)
- 3. **Say**: Calculate the expression on the board for Question a.  $5 \times 100000$ . Please raise your hand when you finish.
- 4. Select a pupil who has raised their hand to answer. (Answer:  $5 \times 100000 = 5 \times 10^{5}$ )
- 5. Ask: What do we call numbers we write in this way? Raise your hand. (Answer: Standard form)
- 6. Say: Today we are going to interpret and write small numbers in standard form.

- 1. Say: Calculate Question b.  $5 \div 1000$ . What is the answer? Raise your hand.
- 2. Select a pupil who has raised their hand to answer. (Answer: 0.005)
- 3. Say: Write Question b. on the board as a power of 10.
- 4. Have a pupil from the back of the classroom volunteer to answer. (Answer: 5.  $\div$  10<sup>3</sup>)
- 5. Write on the board:

	5 ÷ 1000	=	5 ÷ 10 <sup>3</sup>	
Say: We write this as:		=	$\frac{5}{10^{3}}$	
		=		 complete later

- 6. **Say**: We want to write this so we do not have a fraction in our answer. Think back to writing numbers using index notation.
- 7. Ask: What should we do?
- 8. Allow the pupils a few moments to think (less than a minute).
- 9. Ask pupils to pair up with their neighbour. Allow a few more moments for them to discuss how to write the number in index notation.
- 10. Ask: Who would like to share their ideas with the class? Raise your hand.
- 11. Have 2-3 pupils to tell their ideas to the class.
- 12. If no one gives the right answer, guide a pupil to say we use a negative index. (Answer: 5  $\,\times\,\,10^{-3})$
- 13. Complete the answer on the board.

- 14. Do the check for standard form:
  - ✓ Ask: Is 5 equal to or greater than 1? Raise your hand. (Answer: Yes)
  - ✓ Ask: Is 5 less than 10? Raise your hand. (Answer: Yes)
  - ✓ Ask: Is the power of 10 an integer? Raise your hand. (Answer: Yes, it is a negative integer, -3)
  - $\checkmark$  Then the number is in standard form.
- 15. Say: We can change the number from standard form back to an ordinary number.
- 16. Ask: Can someone remind the class what we do to the decimal point when we divide by a power of 10? Raise your hand. (Answer: To divide, move the decimal point to the left by the power of 10. We add zeros in front of the number if necessary.)
- 17. Have a pupil from the front of the classroom volunteer to show this on the board. (Answer:  $5 \times 10^{-3} = \frac{5}{10^3} = \frac{5}{1000} = 0.005$ . Other correct methods are acceptable.)
- 18. **Ask**: If we change back from standard form to the ordinary number, do we get the number we started with? Raise your hand. (Answer: Yes, then we have converted the number correctly to standard form.)
- 19. Write on the board: Change 0.000525 to standard form.
- 20. Work through the question on the board. Have pupils volunteer to answer open-ended questions.

		0.000525	=	$\frac{5.25}{10000}$
21.	Ask: What do we write this as?		=	$\frac{5.25}{10^{4}}$
22.	Ask: What answer do we get?		=	$5.25 \times 10^{-4}$

23. **Say**: In standard form, we write numbers smaller than zero with powers of 10 that are negative integers.

#### **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Point to the questions on the board:

Mark with a  $\checkmark$  if the number is in standard form.

Mark with a **×** if the number is not in standard form. Write the number in standard form.

a. $35.8 \times 10^{-2}$	b.	$7.934 \times 10^{-4}$
c. 0.5278	d.	0.07  imes 10
e. $43.2 \times 10^{-3}$	f.	0.000975

Change these numbers from standard form to ordinary numbers.

- g.  $1.25 \times 10^{-6}$  h.  $1.89 \times 10^{-4}$ i.  $8.5 \times 10^{-7}$  j.  $3.927 \times 10^{-3}$
- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a. ×, 3.58 × 10<sup>-1</sup>; b. ✓; c. ×, 5.278 × 10<sup>-1</sup>; d. 7 × 10<sup>-1</sup>; e. ×, 4.32 × 10<sup>-2</sup>; f. ×, 9.75 × 10<sup>-4</sup>; g. 0.00000125; h. 0.000189; i. 0.0000085; j. 0.003927)

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:

Please change these numbers to standard form:

a.	$87.5 \times 10^{-3}$	b.	$24587 \times 10^{-6}$
c.	0.000008756	d.	0.0000005486
e.	$125 \times 10^{-6}$	f.	0.02

Change these numbers from standard form to ordinary numbers:

g.	$7.25 \times 10^{-6}$	h.	$9 \times 10^{-4}$
i.	$2.173 \times 10^{-3}$	j.	$3.0001 \times 10^{-5}$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a.  $8.75 \times 10^{-2}$ ; b.  $2.4587 \times 10^{-2}$ ; c.  $8.756 \times 10^{-6}$ ; d.  $5.486 \times 10^{-8}$ ; e.  $1.25 \times 10^{-4}$ ; f.  $2 \times 10^{-2}$ ; g. 0.00000725; h. 0.0009; i. 0.002173; j. 0.000030001)

- 1. **Say:** Please write your name on a piece of paper. Now write your answers to the questions on the board. Your work will be collected at the end of the lesson.
- 2. Write on the board: Convert: a. 0.000058 to standard form b.  $6.47 \times 10^6$  to an ordinary number
- 3. Collect the work at the end of the lesson. (Answer: a.  $5.8 \times 10^{-5}$ ; b. 6,470,000)
- 4. Check the work to see how much the pupils have understood so far. Assist pupils in the next lesson when they will be practising converting numbers from standard form to ordinary numbers and vice versa.

Lesson Title: Conversion to and from Standard Form		Theme: Numbers and Numeration	
Lesson Number: M-09-039		Class/Level: JSS 3	Time: 35 minutes
Learning Outcomes	Teaching Aids	<b>Preparation</b>	
By the end of the	None	1. Write on the	e board:
lesson, pupils will be		Convert to sta	ndard form:
able to convert from whole		a. 5,254	b. 0.00589
numbers and decimals to		c. 0.00004781	d. 0.0006
standard form and vice versa.		2. Write the question	is from the Guided
		Practice section on the	ne board.
		3. Write the question	is from the
		Independent Practice	e section on the board.

- 1. Say: Please convert the expressions on the board to standard form:
  - a. 5,254 b. 0.00589
  - c. 0.00004781 d. 0.0006
- 2. Say: Raise your hand when you finish.
- 3. Allow 2 minutes for pupils to answer the questions.
- 4. Select pupils who have raised their hands to answer. (Answers: a.  $5.254 \times 10^3$ ; b.  $5.89 \times 10^{-3}$ ;
  - c.  $4.781 \times 10^{-5}$ ; d.  $6 \times 10^{-4}$ )
- 5. **Say**: Today we are going to convert from whole numbers and decimals to standard form and vice versa.

- 1. **Ask:** Do you notice any pattern between the number of zeros and the power of 10 for the numbers smaller than 0? Raise your hand.
- 2. Select a pupil who raised their hand to answer. (Example answers: The number of zeros is one less than the power of 10; the power of 10 is one more than the number of zeros.)
- **3.** Say: You can use that fact to help when converting to and from standard form.
- 4. Point out to the pupils that we ignore the negative sign when we describe the pattern.
- 5. **Say:** Any number can be written in standard form.
- 6. **Ask:** Who can remind the class the main reason we write numbers in standard form? Raise your hand.
- 7. Select a pupil who has raised their hand. (Answer: They are used to write very large and very small numbers in Maths, Science and everyday life.)
- 6. **Say:** We use standard form if the number is smaller than 0.001 or if it is equal to or greater than 10,000. Otherwise, we write them as ordinary whole numbers or decimal numbers.
- 7. **Say**: Let us look at a practical example. The distance of the Earth from the Sun is approximately 149,600,000,000 metres.
- 8. Write the distance on the board: The distance of the Earth from the Sun is approximately 149,600,000,000 metres.
- 9. Have a pupil volunteer to come to the board to write this distance in standard form. Ask other pupils to work in pairs to write this distance in standard form.
- 10. Check the answer. (Answer:  $1.496 \times 10^{11}$ m)
- 11. Say: The mass of a particle of dust is  $7.53 \times 10^{-10}$  kg.

- 12. Write the mass on the board and ask the pupils to write it as an ordinary number.
- 13. Have a pupil volunteer to come to the board to write the number in standard form on the board. Ask other pupils to work in pairs to do the same.
- 14. Check the answer. (Answer: 0.00000000753 kg)
- 15. **Say**: Remember when converting from standard form to an ordinary number, the power of 10 gives the number of places to the left or right to move the decimal point.

#### **Guided Practice** (10 minutes)

- 1. Ask pupils to continue to work in pairs.
- 2. Point to the questions on the board:

Change these numbers from standard form to ordinary numbers:

a. 2.5 $ imes$ 10 $^{6}$	b.	$3.98 \times 10^{-4}$
c. $5.874 \times 10^{7}$	d.	$9.927 \times 10^{-2}$

Convert these from ordinary numbers to standard form:

e. 0.0087	f.	256,599
g. 8,875,235	h.	0.0000001483

i. The distance that light travels in a year is 5,870,000,000,000 metres. Write this in standard form.

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers on the board. Ask pupils to check their work. (Answers: i. 2,500,000; ii. 0.000398; iii. 58,740,000; iv. 0.09927; v. 8.7 × 10<sup>-3</sup>; vi. 2.56599 × 10<sup>-5</sup>; vii. 8.875235 × 10<sup>-6</sup>; viii. 1.483 × 10<sup>-7</sup>; ix. 5.87 × 10<sup>-12</sup>)

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:

Convert these numbers from standard form to ordinary numbers:

a.  $6.741 \times 10^{9}$  b.  $5.62 \times 10^{-10}$ 

Convert these from ordinary numbers to standard form

- c. 0.00000000254 d. 698,478,000,000,000
- e. The distance of the Earth from the moon is 384,400,000 metres. Write this in standard form:
- f. Which number below has the same value as  $4.8 \times 10^{4}$ ?  $4.8^{3}$   $4.8^{4}$   $(4.8 \times 10)^{4}$   $0.48 \times 10^{3}$   $0.48 \times 10^{5}$
- g. Which numbers below has the same value as  $8.7 \times 10^{-3}$ ?

8700 0.00087 87  $\times 10^{-4}$  -8.7  $\times 10^{3}$  8.7  $\times 10^{3}$ 

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers on the board. Ask pupils to check their work. (Answers: a. 6,741,000,000; b. 0.00000000562; c. 2.54 × 10<sup>-11</sup>; d. 6.98 × 10<sup>14</sup>; e. 3.844 × 10<sup>8</sup> m; f. 0.48 × 10<sup>5</sup>; g. 87 × 10<sup>-4</sup>)

- 1. **Say:** Write one thing or fact you know about converting numbers into standard form in your exercise books. Also write one fact you know about converting from standard form to ordinary number.
- Have pupils from around the classroom volunteer to share one or both facts with the class. (Example answers: Standard form is used to write very large and very small numbers; standard form is written with a whole number or decimal number equal to or larger than 1, but smaller than 10; the index in a number written in standard form is an integer)

Lesson Title: Multiplying and Dividing Small and	Theme: Numbers and Numerat	ion
Large Numbers		
Lesson Number: M-09-040	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	None	1. Write on the board:
lesson, pupils will be		Calculate:
able to:		$10^2 \times 10^3$ $10^5$
1. Do simple multiplication and		a. $10^{-1} \times 10^{3}$ b. $\frac{10^{2}}{10^{2}}$
division problems with whole		c. $(5.1 \times 10^2) \times (7 \times 10^3)$
numbers, decimals and		d. $(9 \times 10^5) \div (3 \times 10^3)$
fractions.		e. $(3 \times 10^{-4}) \times (8 \times 10^{2})$
2. Give answers to problems in		f. $(5.6 \times 10^6) \div (7 \times 10^2)$
standard form.		2. Write the questions from the Guided
		Practice section on the board.
		3. Write the questions from the
		Independent Practice section on the board.

1. Say: Please write down the answers to Questions a. and b. from the board:

a. 
$$10^2 \times 10^3$$
 b.  $\frac{10^5}{10^2}$ 

2. Allow 1 minute for pupils to answer the question.

3. Have pupils volunteer to explain their answer on the board. (Answers: a.  $10^2 \times 10^3 = 10^{2+3} =$ 

10<sup>5</sup>; b. 
$$\frac{10^{5}}{10^{2}} = 10^{5-2} = 10^{3}$$
).

- 4. Correct any errors in the calculation on the board. Ask pupils to check their work.
- 5. **Say**: We use the laws of indices to calculate with numbers written in standard form. Today we are going to do simple multiplication and division problems with whole numbers, decimals and fractions. We will give our answers in standard form.

- 1. **Say:** Write down the answer to Question c:  $(5.1 \times 10^2) \times (7 \times 10^3)$ . Raise your hand when you finish.
- 2. Select a pupil who raised their hand to explain their answer step-by-step on the board.
- 3. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown below.

$$(5.1 \times 10^{2}) \times (7 \times 10^{3}) = 5.1 \times 7 \times 10^{2} \times 10^{3} \quad \text{Collect like terms}$$

$$= 5.1 \times 7 \times 10^{2+3} \quad \text{Apply 1}^{\text{st}} \text{ law of indices}$$

$$= 35.7 \times 10^{5} \quad \text{Multiply 5.1 and 7, add the indices}$$

$$= 3.57 \times 10^{6} \quad \text{Convert answer to standard form}$$

- 4. **Say:** We use the same method for dividing numbers in standard form.
- 5. Show how to calculate the answer to Question d:

$$(9 \times 10^{5}) \div (3 \times 10^{3}) = \frac{9 \times 10^{2}}{3 \times 10^{3}} \qquad \qquad \text{Write as a fraction}$$

$$= \frac{9}{3} \times \frac{10^{5}}{10^{3}} \qquad \qquad \text{Collect like terms}$$

$$= \frac{9}{3} \times 10^{5-3} \qquad \qquad \text{Apply 2}^{nd} \text{ law of indices}$$

$$= 3 \times 10^{2} \qquad \qquad \text{Divide 9 by 3, subtract the indices}$$

6. Ask pupils to work in pairs for 3 minutes to discuss and share ideas for Questions e. and f: e.  $(3 \times 10^{-4}) \times (8 \times 10^{2})$ 

f.  $(5.6 \times 10^6) \div (7 \times 10^2)$ 

- 7. **Say**: Please raise your hand when you finish.
- 8. Select a pupil who has raised their hand to explain their answer to Question e. on the board.
- 9. Correct any errors in the calculation on the board. Ask pupils to check their work.

$(3 \times 10^{-4}) \times (8 \times 10^{2})$	=	$3 \times 8 \times 10^{-4} \times 10^{2}$	Collect like terms
:	=	$3 \times 8 \times 10^{-4+2}$	Apply 1 <sup>st</sup> law of indices
:	=	$24 \times 10^{-2}$	<ul> <li>Multiply 3 and 8, add the indices</li> </ul>
	=	$2.4 \times 10^{-1}$	Convert answer to standard form

- 10. Select another pupil who has raised their hand to explain their answer to Question f. on the board.
- 11. Correct any errors in the calculation on the board. Ask pupils to check their work.

12. Say: We can skip some of the steps once we are confident in answering questions like these.

#### **Guided Practice** (10 minutes)

- 1. Ask pupils to continue to work in pairs.
- 2. Point to the questions on the board:

Please calculate the following. Give your answers in standard form.

y. $(6 \times 10^{7}) \times (5 \times 10^{6})$	z. $(4 \times 10^{-4}) \times (8.2 \times 10^{2})$
aa. $(3 \times 10^{3}) \times (2.1 \times 10^{-2})$	bb. $(3 \times 10^{4}) \div (6 \times 10^{2})$
cc. $(6.2 \times 10^{5}) \div (2 \times 10^{-8})$	dd. $(3 \times 10^{-4}) \div (4 \times 10^{-4})$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers on the board. Ask pupils to check their work. (Answers: a.  $(6 \times 10^{7}) \times (5 \times 10^{6}) = 6 \times 5 \times 10^{7+6} = 30 \times 10^{13} = 3 \times 10^{14}$ ; b.  $(4 \times 10^{-4}) \times (8.2 \times 10^{2}) = 4 \times 8.2 \times 10^{-4+2} = 32.8 \times 10^{-2} = 3.28 \times 10^{-1}$ ; c.  $(3 \times 10^{3}) \times (2.1 \times 10^{-2}) = 3 \times 2.1 \times 10^{3+(-2)} = 6.3 \times 10^{1}$  or  $6.3 \times 10$ ; d.  $(3 \times 10^{4}) \div (6 \times 10^{3}) = 0.5 \times 10^{4-3} = 0.5 \times 10^{1} = 5 \times 10^{0} = 5$ ; e.  $(6.2 \times 10^{5}) \div (2 \times 10^{-8}) = 12.4 \times 10^{5-(-8)} = 12.4 \times 10^{13} = 1.24 \times 10^{14}$ ; f.  $(3 \times 10^{-4}) \div (4 \times 10^{-4}) = 0.75 \times 10^{-4-(-4)} = 0.75 \times 10^{0} = 0.75 = 7.5 \times 10^{-1}$ ).

#### Independent Practice (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Point to the questions on the board:

Calculate the following. Give your answers in standard form.

a. $(3 \times 10^{7}) \times (6 \times 10^{-6})$	b. $(3.3 \times 10^{-4}) \times (1.2 \times 10^{-2})$
c. $(4.2 \times 10^3) \div (6 \times 10^{-2})$	d. $(1.32 \times 10^{4}) \div (4 \times 10^{-5})$
e. $(6.2 \times 10^2) + (2 \times 10^3)$	f. $(3 \times 10^{6})^2$

- 3. Walk around, if possible, to check the answers and clear up any misconceptions.
- 4. Ask pupils to exchange their exercise books and check each other's work.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.

6. Write the correct answers and steps on the board. Ask pupils to check their work. (Answers: a.  $(3 \times 10^{7}) \times (6 \times 10^{-6}) = 3 \times 6 \times 10^{7+(-6)} = 18 \times 10^{-1} = 1.8 \times 10$ ;

b.  $(3.3 \times 10^{-4}) \times (1.2 \times 10^{-2}) = 3.96 \times 10^{-4+(-2)} = 3.96 \times 10^{-6}$ ;

c. 
$$(4.2 \times 10^{3}) \div (6 \times 10^{-2}) = 0.7 \times 10^{3-(-2)} = 0.7 \times 10^{5} = 7 \times 10^{4}$$
;

d. 
$$(1.32 \times 10^{4}) \div (4 \times 10^{-5}) = 0.33 \times 10^{4-(-5)} = 0.33 \times 10^{9} = 3.3 \times 10^{8}$$
;

e.  $(6.2 \times 10^{2}) + (2 \times 10^{3}) = 620 + 2000 = 2620 = 2.6 \times 10^{3}$ ;

f.  $(3 \times 10^{6})^{2} = (3 \times 10^{6}) \times (3 \times 10^{6}) \times = 3 \times 3 \times 10^{6+6} = 9 \times 10^{12})$ 

- 1. Ask: What do you notice about the way we calculated Question e.? Raise your hand.
- 2. Allow pupils a few moments to look at the calculation for Question e. (Answer: Numbers were changed to ordinary numbers, the calculation was done then changed back to standard form.)
- 3. **Ask:** Who can think of a way to calculate  $(3 \times 10^{6})^{2}$  in our heads? Raise your hand. (Answer: Multiply  $3 \times 3 = 9$  and  $6 \times 2 = 12$  to get  $9 \times 10^{12}$ ).
- 4. **Say:** The way we write numbers in standard form makes it easy to do calculations like this in our heads.

Lesson Title: Right-angled Triangles (Revision)	Theme: Geometry		
Lesson Number: M-09-041	Class/Level: JSS 3	Time: 35 minutes	

		A .
Learning Outcomes	Teaching Aids	Preparation
By the end of the	A poster on large-	1. Make a poster on large-sized
lesson, pupils will be	sized paper,	paper, (vanguard if available) of
able to:	(vanguard if available)	the information in the table titled: Parts
1. Identify the parts of a right-	of the information in	and Properties of a Right-angled
angled triangle.	the table titled: Parts	Triangle (see end of lesson plan). If
2. Identify the properties of a	and Properties of a	poster paper is not available, use the
right-angled triangle.	Right-angled Triangle. If	board.
	poster paper is not	2. Draw the triangles at the end of this
	available, use the board.	lesson plan on the board.
		3. Write on the board: Vocabulary List –
		hypotenuse, acute, complementary,
		right-angle
		4. Write the questions from the Guided
		Practice section on the board.

- 1. Ask: What shapes do we have on the board? (Answer: Triangles)
- 2. **Say:** Work in pairs. Write down the triangles that have right angles as one set and those that do not have right angles as another set. (Answer: right angles: { a, b, f }; not right angles: { c, d, e })
- 3. **Say**: Today we are going to identify the parts and properties of a right-angled triangle.

- 1. **Say:** Please work in pairs. Draw a right-angled triangle. Write down everything you know about the triangle.
- 2. Allow pupils 3 minutes to write down everything they know.
- 3. Ask: Who can tell the class what is a right-angled triangle? Raise your hand.
- 4. Select 2-4 pupils to share their ideas of what a right-angled triangle is.
- 5. **Say:** Very good. A right-angled triangle is a triangle which has got a 90° angle. We call 90° a right angle.
- 6. Ask: Who would like to come to the board and draw a right-angled triangle? Raise your hand.
- 7. Select a pupil who has raised their hand to draw a right-angled triangle on the board.
- 8. **Say** (to the pupil at the board): The class is now going to tell you what they know about rightangled triangles. You will show this on your triangle.
- 9. **Say**: Raise your hand and tell (name of pupil) one thing you know about a right-angled triangle.
- 10. Select pupils from around the classroom to share their ideas about right-angled triangles.
- 11. Guide the pupil at the board to write the information down briefly next to the triangle.
- 12. Say: Thank you for all your contributions. Let us put all this information together in a table.
- 13. Show the poster titled Parts and Properties of a Right-angled Triangle/ point to the table on the board.
- 14. Go through the details of the parts and properties of a right-angled triangle. Point to each information as you say it.
- 15. Do not rush through the information. Allow time for the pupils to absorb it. Repeat any information you feel necessary.
- 16. **Say**: Please copy the table in your exercise books.

# **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs to answer the question on the board.
- 2. Point to the questions on the board:

Which side is the hypotenuse in each of the following right-angled triangles?



- 3. Walk around, if possible, to check their answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: a. JK; b. TS; c. PQ; d. YZ)

# Independent Practice (10 minutes)

- 1. Ask the pupils to work independently to answer the questions.
- 2. Write the following questions on the board:

Write 2 similarities and 2 differences between a scalene right-angled triangle and an isosceles right-angled triangle.

- 3. Walk around, if possible, to check their answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give one similarity or difference.
- 5. Correct any errors:

(Answers:

Scalene and Isosceles Right-angled Triangles

Similarities	Differences	
Poth have a 00° angle	Scalene has 3 different angles, isosceles	
Both have a 90° angle	has 2 angles the same size (45°)	
Both have one long side and 2 shorter	Scalene has 3 sides of different lengths;	
sides	Isosceles has 2 sides of same length	
Both have 2 acute angles which are		
complementary angles)		

- 1. Say: Write down 2 different things you learned today in your pairs.
- 2. Allow pupils 1 minute to discuss and share their ideas.
- 3. Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: That there are 2 types of right-angled triangles: scalene and isosceles; an

equilateral triangle cannot be a right-angled triangle; the 2 acute angles in a right-angled triangle are complementary angles. Accept all reasonable answers.)

[TRIANGLES FOR OPENING ACTIVITY]



# [PARTS AND PROPERTIES OF A RIGHT-ANGLED TRIANGLE]

Parts of a right-angled triangle				
hypotenuse	<ul> <li>3 sides</li> <li>longest side, c, is called the hypotenuse</li> <li>2 shorter sides a and b</li> </ul>			
	<ul> <li>3 angles</li> <li>one angle equal to 90°, marked with a square</li> <li>2 other angles smaller than 90° (acute angles)</li> </ul>			
Pro	operties of a Right-Angled Triangle			
scalene right-angled triangle $a \underbrace{c}_{b}$ isosceles right-angled triangle	<ul> <li>all 3 angles add up to 180°</li> <li>the 2 acute angles are also complementary angles – they add up to 90°</li> <li>the 2 short sides, a and b form the 90° angle</li> <li>the hypotenuse, c, is the side opposite the 90° angle</li> <li>either one of the short sides is the base, the other will be the height</li> <li>there are 2 types of right-angled triangles: scalene and isosceles</li> <li>scalene right-angled triangle</li> </ul>			
	<ul> <li>all 3 sides are different from each other</li> <li>one 90° angle, 2 other angles different from each other</li> <li>isosceles right-angled triangle</li> <li>have 2 equal sides which form the 90° angle</li> <li>the 2 angles are 45° each</li> <li>a right angle can never be equilateral as the hypotenuse will always be longer than the other 2 sides</li> </ul>			

Pupils can be asked to create their own versions of this poster as a class activity. The best posters can be put up on the wall of the classroom.

Lesson Title: Introduction to Pythagoras' Theorem		Theme: Geometry		
Lesson Number: M-09-042		Class/Level: JSS 3	Time: 35 minutes	
Lesson Number: M-09-042	<b>Teaching Aids</b> 1. Table of Squ Roots found at lesson plan. This wil	ares and Square the end of this l be used for all	Preparation 1. Create a poster showing the simplified statement of Pythagoras'	
<ol> <li>State Pythagoras' theorem.</li> <li>Identify that the formula a<sup>2</sup> + b<sup>2</sup> = c<sup>2</sup> can be used to find the sides of a right- angled triangle.</li> </ol>	lessons on Pythagor 2. A poster showing statement of Pytha on vanguard paper, use the board (see plan)	as' Theorem. g the simplified goras' Theorem if available. Not, end of lesson	Theorem on vanguard paper, if available. Not, use the board (see end of lesson plan) 2. Draw the right-angled triangle below on the board. 3 cm	
			<i>4 cm</i> 3. Draw the table of Squares and Square Roots found at the end of this lesson plan, on the board (select appropriate sections to write as needed in the lesson).	

- 1. **Ask:** What best describes the type of triangle on the board? Raise your hand. (Answer: scalene right-angled triangle)
- 2. **Say**: Please copy the triangle into your exercise books. Use your ruler to measure the sides accurately. Mark the hypotenuse and the right angle on the triangle.
- 3. Allow a few moments for pupils to do so.
- 4. **Say**: Today we are going to state and use Pythagoras' Theorem to find the sides of a right-angled triangle.

# Introduction to the New Material (10 minutes)

- 1. **Say:** Pythagoras' Theorem links the length of the hypotenuse of a rightangled triangle to the lengths of the other 2 sides.
- 2. Ask pupils to verify that the 3 sides of their triangles measure 3, 4 and 5 centimetres each.

This will be the case depending on how accurately they drew the triangle.

- 3. Write on the board:  $3^2 + 4^2 = 5^2$
- 4. **Say**: We are going to verify that the sides of our triangle are linked by this equation.
- 5. The following explanation shows Pythagoras' Theorem on a practical level that will help pupils understand the definition better.
- 6. Draw the squares and complete the calculations as they give the answers to your questions.



- 7. Ask pupils to draw a square on the side measuring 3 cm.
- 8. Ask: Can someone tell us what is the area of this square? Raise your hand. (Answer:  $3 \times 3 = 9 \text{ cm}^2$ )
- 9. Ask pupils to draw a square on the side measuring 4 cm. Ask them to calculate its area. (Answer:  $4 \times 4 = 16$  cm<sup>2</sup>)
- 10. Say: Add the 2 areas, what do we get? (Answer:  $9 + 16 = 25 \text{ cm}^2$ )
- 11. Ask pupils to draw a square on the side measuring 5 cm (hypotenuse). Ask them to calculate its area. (Answer:  $5 \times 5 = 25$  cm<sup>2</sup>)
- 12. The full calculation is shown below:

$$3^{2} + 4^{2} = 9 + 16$$
  
= 25  
= 5<sup>2</sup>

- 13. Say: We have just proved Pythagoras' Theorem.
- 14. **Say**: The full statement of Pythagoras' Theorem is:
  - It is true for all right-angled triangles. 'Pythagoras' Theorem states that for any right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other 2 sides.'
- 15. Explain how this statement applies to our triangle.
- 16. Point to the square on the hypotenuse of our triangle and show how its area is equal to the sum of the areas of the other 2 sides. You may need to repeat this explanation more than once.
- 17. Say: The full statement of Pythagoras' Theorem can be simplified so it is easier to remember.
- 18. Put up the poster or point to the Pythagoras' Theorem on the board.
- 19. **Say**: If we know the length of any 2 sides of a right-angled triangle, we can find the length of the missing side using Pythagoras' Theorem.
- 20. Ask pupils to copy the simplified statement in their exercise books.

#### **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. Write on the board:

Please verify Pythagoras' Theorem for the triangles below:



- 3. Point to the squares and square roots tables found at the back of this book.
- 4. Walk around, if possible, to check their answers and clear up any misconceptions.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers and steps on the board. Ask pupils to check their work.

Answers: a. a = 5, b = 12, c = 13  $a^2 + b^2 = c^2$ b. a = 9, b = 12, c = 15  $a^2 + b^2 = 5^2 + 12^2$  = 25 + 144 = 169  $a^2 + b^2 = c^2$  b. a = 9, b = 12, c = 15  $a^2 + b^2 = 9^2 + 12^2$ = 81 + 144

$$c^{2} = 13^{2}$$
  
 $= 169$   
 $5^{2} + 12^{2} = 13^{2}$   
LHS = RHS  
right-angled triangle  
 $c^{2} = 15^{2}$   
 $= 225$   
 $9^{2} + 12^{2} = 15^{2}$   
LHS = RHS  
right-angled triangle

#### **Independent Practice** (10 minutes)

- 1. Ask pupils to work independently to answer the questions.
- 2. Write the following questions on the board:

Verify Pythagoras' Theorem for the triangles below:



- 3. Point to the squares and square roots tables on the board.
- 4. Walk around, if possible, to check their answers and clear up any misconceptions.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. **Write** the correct answers and steps on the board. Ask pupils to check their work. Answers:

$$a^2 + b^2 = c^2$$

a. a = 10, b = 24, c = 26b. a = 8, b = 15, c = 16 $a^{2} + b^{2} = 10^{2} + 24^{2}$ = 100 + 576 = 676  $a^{2} + b^{2} = 8^{2} + 15^{2}$ = 64 + 225 289 = 676 289  $c^2 = 26^2$  $c^2 = 16^2$ = 676 = 256  $10^2 + 24^2 = 26^2$  $8^2 + 15^2 \neq 16^2$ LHS = RHSLHS  $\neq$  RHS not a right-angled triangle, right-angled triangle c should be equal to 17

- 1. Say: Please write down one new thing you learned today.
- 2. Allow pupils 1 minute to discuss and share their ideas.
- 3. Have one pupil from around the classroom volunteer to answer. (Example answers: How to find the link between the length of the hypotenuse of a right-angled triangle and the lengths of the other 2 sides; how to verify Pythagoras' Theorem)



<u>Pythagoras' Theorem</u> states that for any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other 2 sides.

If we use *a*, *b* and *c* for the sides of the triangle as shown, then:



Lesson Title: Finding the Hypotenuse of a Right-	Theme: Geometry	
Angled Triangle		
Lesson Number: M-09-043	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes 1. By the end of the lesson, pupils will be able to find the hypotenuse of a right- angled triangle using Pythagoras' theorem.	Teaching Aids Table of Squares and Square Roots found at the end of this lesson plan. This will be used for all lessons on Pythagoras' Theorem.	<ul> <li>Preparation <ol> <li>Draw the triangles for this lesson plan on the board (see end of this lesson plan).</li> </ol> </li> <li>Note: Do not write the answers until after pupils have answered the questions. <ol> <li>Draw the table of Squares and Square Roots found at the end of this lesson plan, on the board (select appropriate sections to write as needed in the lesson).</li> <li>Write the questions at the end of the properties and the end of the properties and the end of the properties at the end</li></ol></li></ul>
		lesson plan, on the board.

- 1. Ask: Which equation gives Pythagoras' Theorem for the triangles in Question 1?
- 2. Allow pupils to work in pairs for 2 minutes to answer the question.
- 3. Have a pupil volunteer to explain their answer on the board.
- 4. Correct any errors in the calculation on the board. Ask pupils to check their work. (Answers: see end of this lesson plan)
- 5. **Say**: Today we are going to find the hypotenuse of a right-angled triangle using Pythagoras' theorem.

- 1. **Say:** In the last lesson, we used Pythagoras' Theorem to verify if a triangle is a right-angled triangle. Its main use is to find the lengths of the hypotenuse or one of the shorter sides.
- 2. Ask: Who can remind the class of what Pythagoras' Theorem states? Raise your hand. (Answer:  $a^2 + b^2 = c^2$ )
- 3. **Say:** Let us use an adjusted version to answer Question 2. We should always draw the triangle in questions such as these so we can be sure we are finding the length of the correct side. Please draw the triangle in your exercise books.
- 4. Allow time for pupils to draw the triangles in their exercise books.
- 5. Show the calculation on the board. Explain each line of the calculation.
- 6. Ask pupils to give answers for the steps in the calculations.

7. Say: By Pythagoras' Theorem:	$c^2$	=	$a^2 + b^2$
8. Ask: What are the values for a and b?			
(Answer: $a = 6 \text{ cm}, b = 8 \text{ cm}$ )		=	$6^2 + 8^2$
9. <b>Ask:</b> What is $6^2$ ? (Answer: 36)		=	36 + 64
10. <b>Ask:</b> What is 8 <sup>2</sup> ? (Answer: 64)			00 1 01
11. Ask: What do we get when we add 36 to 64?	)	=	100
12. Ask: How do we find c?			
(Answer: Take the square root of 100)	С	=	$\sqrt{100}$
13. Ask: What is the square root of 100?			
(Answer: 10)	С	=	10 cm

- 14. **Say**: Some of you may be thinking that  $\sqrt{100} = \pm 10$ . That is correct. Since we cannot have a negative measurement for length we only take the positive square root. Please do Question 3 in your exercise books. Check your answer with your neighbour when you are finished.
- 15. Allow pupils time to do Question 3.
- 16. Ask: Who would like to explain Question 3 on the board? Raise your hand.
- 17. Select a pupil who has raised their hand to explain their answer step-by-step on the board.
- 18. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown below.



# **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. **Say**: Please calculate the length of the hypotenuse of the triangles for Questions 4 and 5 on the board.
- 3. Point to the squares and square roots tables on the board.
- 4. Walk around, if possible, to check their answers and clear up any misconceptions.
- 5. Have pupils from around the classroom volunteer to give their answers to the questions.
- 6. Write the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

#### **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer Questions 6, 7, 8 and 9.
- 2. Point to the squares and square roots tables on the board.
- 3. Walk around, if possible, to check their answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to Questions 6 and 9.
- 5. Do not provide the answers for Questions 7 and 9. Use them to check pupils' understanding of the work.
- 6. Write the correct answers for Questions 6 and 9 on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

- 1. **Say:** Please write your name on a piece of paper. Now write your working out and answer for Questions 7 and 9 on the paper. Hand in the paper in at the end of the lesson.
- 2. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be applying Pythagoras' theorem to find the length of the other 2 sides of a right-angled triangle.

#### [QUESTIONS FOR OPENING ACTIVITY]

1. Which of these equations show Pythagoras Theorem? A.  $x^{2} = y^{2} + z^{2}$ B.  $y^{2} = x^{2} + z^{2}$ z C.  $z^{2} = x^{2} + y^{2}$  $A.a^2 = b^2 + c^2$ a. b.  $p^2 = q^2 + c.$ A.  $r^2$  $B.b^2 = a^2 + c^2$ р  $q^2 = p^2 + x /$  $C.c^2 = b^2 + a^2$ В. а  $r^2$  $C.r^2 = p^2 +$ q b

(Answers: a. C; b. B; c. A)

## [QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Calculate the length of the hypotenuse in the following triangles:



#### [QUESTIONS FOR GUIDED PRACTICE]

Calculate the length of the hypotenuse in the following triangles:



#### [QUESTIONS FOR INDEPENDENT PRACTICE]

Calculate the length of the hypotenuse in the following triangles. Give answers, if required, to 1 decimal place.



## [TABLES OF SQUARES AND SQUARE ROOTS]

number	square	cube square roo	
n	n²	n <sup>3</sup>	√n
1	1	1	1.0000
2	4	8	1.4142
3	9	27	1.7321
4	16	64	2.0000
5	25	125	2.2361
6	36	216	2.4495
7	49	343	2.6458
8	64	512	2.8284
9	81	729	3.0000
10	100	1000	3.1623
11	121	1331	3.3166
12	144	1728	3.4641
13	169	2197	3.6056
14	196	2744	3.7417
15	225	3375	3.8730
16	256	4096	4.0000
17	289	4913	4.1231
18	324	5832	4.2426
19	361	6859	4.3589
20	400	8000	4.4721
21	441	9261	4.5826
22	484	10648	4.6904
23	529	12167	4.7958
24	576	13824	4.8990
25	625	15625	5.0000
26	676	17576	5.0990
27	729	19683	5.1962
28	784	21952	5.2915
29	841	24389	5.3852
30	900	27000	5.4772
31	961	29791	5.5678
32	1024	32768	5.6569
33	1089	35937	5.7446
34	1156	39304	5.8310
35	1225	42875	5.9161
36	1296	46656	6.0000
37	1369	50653	6.0828
38	1444	54872	6.1644
39	1521	59319	6.2450
40	1600	64000	6.3246

# Squares - Cubes - Square Root (

number	square	cube	square root
n	n <sup>2</sup>	n <sup>3</sup>	.√n
41	1681	68921	6.4031
42	1764	74088	6.4807
43	1849	79507	6.5574
44	1936	85184	6.6332
45	2025	91125	6.7082
46	2116	97336	6.7823
47	2209	103823	6.8557
48	2304	110592	6.9282
49	2401	117649	7.0000
50	2500	125000	7.0711
51	2601	132651	7.1414
52	2704	140608	7.2111
53	2809	148877	7.2801
54	2916	157464	7.3485
55	3025	166375	7.4162
56	3136	175616	7.4833
57	3249	185193	7.5498
58	3364	195112	7.6158
59	3481	205379	7.6811
60	3600	216000	7.7460
61	3721	226981	7.8102
62	3844	238328	7.8740
63	3969	250047	7.9373
64	4096	262144	8.0000
65	4225	274625	8.0623
66	4356	287496	8.1240
67	4489	300763	8.1854
68	4624	314432	8.2462
69	4761	328509	8.3066
70	4900	343000	8.3666
71	5041	357911	8.4261
72	5184	373248	8.4853
73	5329	389017	8.5440
74	5476	405224	8.6023
75	5625	421875	8.6603
76	5776	438976	8.7178
77	5929	456533	8.7750
78	6084	474552	8.8318
79	6241	493039	8.8882
80	6400	512000	8.9443

# Squares - Cubes - Square Root (

number n	square n <sup>2</sup>	cube n <sup>3</sup>	square root √n
81	6561	531441	9.0000
82	6724	551368	9.0554
83	6889	571787	9.1104
84	7056	592704	9.1652
85	7225	614125	9.2195
86	7396	636056	9.2736
87	7569	658503	9.3274
88	7744	681472	9.3808
89	7921	704969	9.4340
90	8100	729000	9.4868
91	8281	753571	9.5394
92	8464	778688	9.5917
93	8649	804357	9.6437
94	8836	830584	9.6954
95	9025	857375	9.7468
96	9216	884736	9.7980
97	9409	912673	9.8489
98	9604	941192	9.8995
99	9801	970299	9.9499
100	10000	1000000	10.0000
101	10201	1030301	10.0499
102	10404	1061208	10.0995
103	10609	1092727	10.1489
104	10816	1124864	10.1980
105	11025	1157625	10.2470
106	11236	1191016	10.2956
107	11449	1225043	10.3441
108	11664	1259712	10.3923
109	11881	1295029	10.4403
110	12100	1331000	10.4881
111	12321	1367631	10.5357
112	12544	1404928	10.5830
113	12769	1442897	10.6301
114	12996	1481544	10.6771
115	13225	1520875	10.7238
116	13456	1560896	10.7703
117	13689	1601613	10.8167
118	13924	1643032	10.8628
119	14161	1685159	10.9087
120	14400	1728000	10.9545

number	square	cube	square root					
n	n <sup>2</sup>	n <sup>3</sup>	$\sqrt{n}$					
121	14641	1771561	11.0000					
122	14884	1815848	11.0454					
123	15129	1860867	11.0905					
124	15376	1906624	11.1355					
125	15625	1953125	11.1803					
126	15876	2000376	11.2250					
127	16129	2048383	11.2694					
128	16384	2097152	11.3137					
129	16641	2146689	11.3578					
130	16900	2197000	11.4018					
131	17161	2248091	11.4455					
132	17424	2299968	11.4891					
133	17689	2352637	11.5326					
134	17956	2406104	11.5758					
135	18225	2460375	11.6190					
136	18496	2515456	11.6619					
137	18769	2571353	11.7047					
138	19044	2628072	11.7473					
139	19321	2685619	11.7898					
140	19600	2744000	11.8322					
141	19881	2803221	11.8743					
142	20164	2863288	11.9164					
143	20449	2924207	11.9583					
144	20736	2985984	12.0000					
145	21025	3048625	12.0416					
146	21316	3112136	12.0830					
147	21609	3176523	12.1244					
148	21904	3241792	12.1655					
149	22201	3307949	12.2066					
150	22500	3375000	12.2474					
151	22801	3442951	12.2882					
152	23104	3511808	12.3288					
153	23409	3581577	12.3693					
154	23716	3652264	12.4097					
155	24025	3723875	12.4499					
156	24336	3796416	12.4900					
157	24649	3869893	12.5300					
158	24964	3944312	12.5698					
159	25281	4019679	12.6095					
160	25600	4096000	12.6491					

Lesson Title: Finding the Other Sides of a Right-	Theme: Geometry	
Angled Triangle		
Lesson Number: M-09-044	Class/Level: JSS 3	Time: 35 minutes

Learning Outcomes	Teaching Aids	<b>Preparation</b>
By the end of the	Table of Squares and	1. Draw the triangles for this
lesson, pupils will	Square Roots (see end of	lesson plan on the board (see
be able to apply	lesson plan). This will be used	end of lesson plan). Note: Do not
Pythagoras' theorem to find	for all lessons on Pythagoras'	write the answers until after pupils
the length of the other 2	Theorem.	have answered the questions.
sides of a right-angled		2. Draw the Table of Squares and
triangle.		Square Roots on the board.
		3. Write the questions at the end of
		the lesson plan, on the board.

- 1. **Say**: Find the hypotenuse for the triangle in Question A. Raise your hand when you have the answer.
- 2. Allow 2 minute for pupils to answer the question.
- 3. Have pupils volunteer to explain their answer on the board. (Answers: see end of this lesson plan)
- 4. **Say**: Today we are going to apply Pythagoras' theorem to find the length of the other 2 sides of a right-angled triangle.

- 1. Ask a pupil to read Question B on the board.
- 2. **Say**: We will use Pythagoras' Theorem to find the length of the side.
- 3. Show on the board how to use Pythagoras' Theorem to find the area of the side. Ask pupils to raise their hand to answer questions.

4.	Say: By Pythagoras' Theorem:	$a^2 + b^2$	=	c <sup>2</sup>	
5.	<b>Ask:</b> What do we know about this triangle?	1 <b>-</b> 2 + <b>b</b> 2	_	172	•
	(Answer: $a = 15 \text{ cm}, c = 17 \text{ cm}$ )	$15^{-} + 0^{-}$	=	1/-	$h \wedge i=$
6.	Ask: What is $15^2$ ? (Answer: 225)	225 - h2	_	200	~~
7.	<b>Ask:</b> What is 17 <sup>2</sup> ? (Answer: 289)	$225 \pm 0^{-1}$	-	209	$\leq$
8.	Say: We now subtract 225 from both sides.	b <sup>2</sup>	=	289 – 225	17 cm
9.	Ask: What do we get when we do that?		=	64	
10.	Ask: What do we do next?	,		<u> </u>	
	(Answer: Take the square root of 64)	b	=	√64	
11.	Ask: What does that give us? (Answer: 8)		=	8 cm	



- 12. Say: Please do Question C in your $a = \sqrt{81}$ exercise books. Check your answera = 9 cmwith your neighbour when you finish.a = 9 cm
- 13. Allow pupils time to do Question C
- 14. **Ask**: Who would like to explain Question C on the board? Raise your hand.
- 15. Select a pupil who has raised their hand to explain their answer step-by-step on the board.
- 16. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown above.

# **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. **Say**: Please calculate the length of the missing side of the triangles for Questions D and E. You will need the table which gives the squares and square roots.
- 3. Walk around, if possible, to check their answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. **Write** the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan).

## **Independent Practice** (10 minutes)

- 1. Ask the pupils to work independently to answer Questions F and G.
- 2. Walk around, if possible, to check their answers and clear up any misconceptions.
- 3. Have pupils from around the classroom volunteer to give their answer to the question F.
- 4. Write the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)
- 5. Do not discuss the answer for Question G. Use them to check pupils' understanding of the work.

- 1. **Say:** Please write your name on a piece of paper. Now write your working out and answer for Questions G. on the paper. Hand the paper in at the end of the lesson.
- 2. Check the work done by the pupils after the lesson. Use it as a guide to which pupils need additional assistance during the next lesson when pupils will be solving diagram and word problems involving Pythagoras' theorem.

[QUESTION FOR THE OPENING ACTIVITY]

A. Calculate the length of the hypotenuse in the following triangle. Give your answer to 1 decimal place.



#### [QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

Calculate the length of the missing side in the following triangles:



#### [QUESTIONS FOR GUIDED PRACTICE]

D.

Calculate the length of the missing side in the following triangles. Give your answer to 1 decimal place, if required.



#### [QUESTIONS FOR INDEPENDENT PRACTICE]

Calculate the length of the missing side in the following triangles. Give your answer to 1 decimal place, if required.

F. Answer G. Answer  $a^2 + b^2$  $a^{2} + b^{2}$  $= c^{2}$  $= c^{2}$ 10 cm 25<sup>2</sup>  $10^2 + b^2$  $a^2 + 18^2$ 26<sup>2</sup> = 625  $100 + b^2$ 676 25 cm  $a^2 + 324$ = 18 cm 26 cm b<sup>2</sup> = 625 - 324 = 676 - 100  $b^2$ 576  $a^2$ 301 = = b а  $\sqrt{301}$  $\sqrt{576}$ = а = b 24 cm 17.3 cm = = а

Lesson Title: Applying Pythagoras' Theorem	Theme: Geometry	
Lesson Number: M-09-045	Class/Level: JSS 3	Time: 35 minutes

	Learning Outcomes	Teaching Aids	<b>Preparation</b>
	By the end of the	Table of Squares	1. Draw the triangles for this
	lesson, pupils will be	and Square Roots.	lesson plan on the board (see
able to	o solve diagram and word	This will be used for	end of this lesson plan).
proble	ems involving Pythagoras'	all lessons on	2. Write the questions at the end of the
theore	em.	Pythagoras' Theorem.	lesson plan on the board.

- 1. **Say**: We have been solving problems on Pythagoras' Theorem for the last few lessons. We have been using fairly simple diagrams so far. Before we do any more problems, let us check that we can all identify the parts of a right-angled triangle.
- 2. **Ask**: How do we identify the hypotenuse in a right-angled triangle? Raise your hand. (Answer: It is the side opposite the right angle.)
- 3. **Ask**: What do we call a right-angled triangle which has 2 equal sides and angles? Raise your hand. (Answer: isosceles right-angled triangles)
- 4. Ask: What size is the angle in an isosceles right-angled triangle? Raise your hand. (Answer: 45°)
- 5. Say: Today we are going to solve diagram and word problems involving Pythagoras' theorem.

- 1. Have a pupil volunteer to read Question a. on the board.
- 2. Ask: Who can tell the class what the question is asking us to do? Raise your hand.
- 3. Select a pupil who has raised their hand to answer. (Answer: To find the length of the diagonal in the rectangle.)
- 4. **Say:** We want to find the diagonal of the rectangle. We know the length of the sides. Let us draw the diagonal to see if that helps us.
- 5. Draw the diagonal on the rectangle.
- 6. Ask: Do you recognise the shape we now have?
- 7. Guide a pupil to say what the shape now looks like. (Answer: 2 right angled triangles)
- 8. **Ask**: How can we find the length of the diagonal? Raise your hand. (Answer: By Pythagoras' Theorem)
- 9. Ask: Who would like to show the calculation on the board? Raise your hand.
- 10. Select a pupil who raised their hand to explain the calculation. Ask other pupils to solve it in their exercise books.
- 11. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown below.



= 19.2 cm to 1 decimal place

- 12. **Say**: You see from this problem how important it is to draw a diagram to help with solving a problem. The diagram showed us that we needed to use Pythagoras' Theorem to solve the problem.
- 13. Ask a pupil to read Question b. from the board.
- 14. **Say:** We do not have a diagram for this problem. Who would like to explain what our first step should be?
- 15. Guide a pupil to say we should draw a diagram.
- 16. If a pupil gives the first step as a calculation using Pythagoras' Theorem, **say**: There is a step before that one which will help us decide whether we are calculating the hypotenuse or one of the shorter sides.
- 17. Show how to interpret the question as a diagram (shown below).
- 18. Have a pupil volunteer to show this on the board and solve the problem. Ask other pupils to solve it in their exercise books.
- 19. Correct any errors in the calculation on the board. Ask pupils to check their work. An example calculation is shown below.



# **Guided Practice** (10 minutes)

- 1. Ask pupils to work in pairs.
- 2. **Say**: Solve for the missing side in Questions c. and d. on the board. You will need the table which gives the squares and square roots.
- 3. Walk around, if possible, to check their answers and clear up any misconceptions.
- 4. Have pupils from around the classroom volunteer to give their answers to the questions.
- 5. Write the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

# Independent Practice (10 minutes)

- 1. Ask the pupils to work independently to answer Questions e. and f. to find the missing sides of the triangles.
- 2. Walk around, if possible, to check their answers and clear up any misconceptions.
- 3. Have pupils from around the classroom volunteer to give their answers to the questions.

4. Write the correct answers on the board. Ask pupils to check their work. (Answers: see the end of this lesson plan)

#### **Closing** (2 minutes)

- 1. Say: Please write down in pairs 2 different things you learned today.
- 2. Allow pupils 1 minute to discuss and share their ideas.
- Have one pupil from the front, and one from the back of the classroom volunteer to answer. (Example answers: How to solve problems with Pythagoras' theorem; solving word problems; it is important to draw a diagram to help with solving a problem)

## [QUESTIONS FOR INTRODUCTION TO THE NEW MATERIAL]

1. Find the length of the diagonal on a rectangle with sides using Pythagoras' Theorem. Give your answer to 1 decimal place.



2. A ladder of length 7.2 m leans against a wall so that the top of the ladder is 7 m above ground level. How far is the bottom of the ladder from the wall? Give your answer to 1 decimal place.

## [QUESTIONS FOR GUIDED PRACTICE]

- 3. An isosceles triangle has a base of 12 cm and a height of 8 cm. Calculate the length, x, of one of its equal sides. (Answer:  $a^2 + b^2 = x^2$ ,  $a = \frac{12}{2}$ , b = 8,  $6^2 + 8^2 = x^2$ ,  $36 + 64 = x^2 = 100$ ,  $x = \sqrt{100} = 10$  cm
- 4. A box measures 60 cm long and 45 cm high. What is the diagonal length of the box to the nearest cm? (Answer:  $a^2 + b^2 = c^2$ , a = 65, b = 50,  $65^2 + 50^2 = c^2$ ,  $4225 + 2500 = c^2$ ,  $c^2 = 6725$ ,  $x = \sqrt{6725} = 82.0$  cm

#### [QUESTIONS FOR INDEPENDENT PRACTICE]

5. Calculate the length of the missing sides in the diagram shown. Give your answers to 1 decimal place if required. (Answer:  $y^2 + 5^2 = 15^2$ ,  $y^2 + 25 = 225$ ,  $y^2 = 225 - 25$ ,  $y^2 = 200$ ,  $y = \sqrt{200} = 14.1$  cm;  $15^2 + 12^2 = x^2$ ,  $225 + 144 = x^2$ ,  $x^2 = 369$ ,  $x = \sqrt{369} = 19.2$  cm







6. A ladder of length 6.5 m leans against a wall so that the bottom of the ladder is 2 m away from the wall. How far is the top of the ladder from ground level? Give your answer to 1 decimal place. (Answer:  $b^2 + 2^2 = 6.5^2$ ,  $b^2 + 4 = 42.25$ ,  $b^2 = 42.25 - 4$ ,  $b^2 = 38.25$ ,  $b = \sqrt{38.25} = 6.2$  m



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# Appendix II: Sines of Angles

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Sines of Angles (x in degrees)

# Appendix III: Cosines of Angles

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  | 1001  | Ŀ                                      | -698  | -673  | -633  
   | -619<br>-606<br>-592   | 583  | 519<br>504  
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| x         0         1         2         4         6         7         6         7         6         7         6         7         6         7         6         7         7         0         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1         2         1  
  |   | é                                      | 178   | 5743<br>613<br>481  | 347   
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| x         a         x         a         x         a         1         x         a         1         x         a         1         x         a         1         x         a         x  
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  |   | 4                                      | 702   | 6505  | 6374  
   | 623  | 567  | 524(  
  | 493  | 447  | 444  | -384  | -335   | 319  
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| x         4         -1         -2         -3         -4         -5         -5         -5         -7         -9         -1         -3         -3         -1         -3         -3         -1         -3 <td>125</td> <td>9</td> <td>034</td> <td>521</td> <td>388</td> <td>252<br/>115<br/>976</td> <td>835<br/>693<br/>548</td> <td>402 255 255 105</td> <td>955</td> <td>802<br/>648<br/>493</td> <td>1179</td> <td>8559<br/>3697<br/>3535</td> <td>1758</td> <td>2206<br/>2040<br/>2874</td> <td>2538<br/>2538<br/>2368</td> <td>2198 2028 1857</td> <td>1685</td> <td>1513</td> <td>0993<br/>0819<br/>0645</td> <td>0471<br/>0297<br/>0122</td> <td>ů</td>   
  | 125   | 9                                      | 034   | 521   | 388   
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  |   | -                                      | 7059  | 6807<br>6678<br>6547  | 6414  
   | 6280<br>6143<br>6004   | 5864<br>5721<br>5577   | 5432<br>5284<br>5135  
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   | 0.2756   | 0-2250  | 0-1736  | 0-1564<br>0-1392<br>0-1219   | 0-1045<br>0-0872<br>0-0698   | 0-0523<br>0-0349<br>0-0175  | ę  
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| X         4         -5         -6         -7         -3         -4         -5         -6         -7         -5         -6         -7         -5         -7         -7         -5         -7 <td>1.91</td> <td>363</td> <td>00</td> <td>000</td> <td>0</td> <td></td> <td></td> <td></td> <td>-</td> <td>- ~ ~</td> <td>+ 10 00</td> <td>000</td> <td>•</td> <td>- 40</td> <td>400</td> <td>N 00 07</td> <td>9</td> <td>TNO</td> <td>400</td> <td>682</td> <td></td>  
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| x         0         ·1         2         3           1         10000         1-000         1-000         1-000         1-000           1         0         1-0000         1-000         1-000         1-000         1-000           2         0-9998         9998         9998         9993         9993         9997         9997           4         0-9976         9974         9973         9972         9997         9997         9973         9972         9997         9997         9997         9997         9974         9973         9973         9972         9997         9997         9997         9974         9973         9973         9973         9973         9973         9973         9973         9973         9973         9974         9973         9974         9779         9778         9744         9779         9744         9779         9774         9779         9744         9779         9744         9779         9774         9779         9774         9779         9779         9744         9779         9774         9779         9774         9779         9744         9719         9714         9719         9744         9719         9719         974   
  | SUBTRACT Differ   | .5 .6 .7 .8 .9 1 2 3 4 5 6             | 6666° 6666° 6666° 6666° 000°L (                       | • .9391939693959395         .939593959395           • .9391939093919397         0         0         0         0           • .9391939093979397         0         0         0         0         0   | 1         -3963         -3966         -3965         -3963         0         0         1         1         1           5         -3954         -3951         -3949         -9947         0         0         1         1         1         1           5         -3954         -3951         -3949         -9947         0         0         1 <td< td=""><td>8 3936 3934 3932 3930 5928 0 0 1 1 1 1 2<br/>7 3914 5912 3910 3907 5905 0 0 1 1 1 1 2<br/>8 3939 3988 3985 3982 3880 0 1 1 1 1 2</td><td>5 - 3823 - 3829 - 3828 - 3823 - 3820 0 1 1 1 2 2</td><td>8 9799 9796 9792 9785 9785 0 1 1 1 2 2 1 1 2 2 1 9763 9755 9751 9748 0 1 1 2 2 2 3 9724 9715 9711 9707 0 1 1 2 2 2</td><td>8. 3681. 3677 3673 3668 3664 0 1 1 2 2 3<br/>1. 3638 3632 3627 3622 3617 0 1 1 2 2 3<br/>0 5500 5632 3627 3522 3617 0 1 1 2 2 3<br/>0 5500 5601 3670 3573 5558 0 1 1 2 2 3</td><td>2 9537 9532 9527 9521 9516 1 1 2 2 3 3 9 9453 9459 9475 9476 9472 9466 9461 1 1 2 2 3 3 9 9453 9471 9415 9470 9446 9461 1 1 2 2 3 3 3</td><td>3 9367 9361 9354 9348 9348 9342 1 1 2 2 3 4 4</td><td>1.3304.9238.9239.9235.9278         1         1         2         3         4         4           5.3239.9222.9225.9219.9212         1         1         2         3         3         4         6           3.347.9150.9143         1         1         2         3         3         4         6</td><td>7 .9100 .9022 .9085 .9078 .9070 1 1 2 3 4 4 5<br/>3 .9026 .9018 .9011 .9003 .8996 1 2 2 3 4 5 5<br/>7 .8949 .8942 .8934 .8926 .8918 1 2 2 3 4 5 5</td><td>8.8870 8862 8854 8846 8838 1 2 2 3 4 5 6 6 5 9 2 9 2 9 2 3 4 5 6 6 5 9 2 9 2 9 2 1 2 2 3 4 5 6 6 9 2 9 2 9 2 9 2 9 2 9 2 9 2 9 2 9 2</td><td>5 .8616 .8607 .8599 .8590 .8581 1 2 3 4 4 5</td><td>6         85.26         86.17         86.06 
       94.99         84.90         1         2         3         4         5         5         6           3         84.34         84.25         8416         83366         1         2         3         4         5         5         6           8         8233         8329         6330         63310         63300         1         2         3         4         5         6         7</td><td>1.8241.8231.8221.8211.8202 1 2 3 4 5 6 7<br/>1.8141.8131.8121.8111.8100 1 2 3 4 5 6 7<br/>0.0001.0001.0002 0002 0002 0001 7 2 3 4 5 6 7</td><td>e 17934 -7934 -7916 -7906 -7891 - 7 2 3 4 5 6 6 - 7826 - 7793 -7793 -7793 -7793 - 7792 - 7793 - 7792 - 7793 - 7792 - 7792 - 7792 - 7792 - 7792 - 7792 - 7792 - 7792 - 779</td><td>7 .7716 .7709 .7694 .7639 .7670 .7569 1 2 3 5 6 7 5 5 7 6 7</td><td>1         7.490         7.448         7.455         7.453         1         2         3         5         6         7           5         7.737         7.337         7.335         1         2         4         5         6         7           6         77245         7.733         7.335         7.335         1         2         4         5         6         7           6         77245         77242         77243         77248         77206         1         2         4         5         6         7</td><td>5 .7133 .7120 .7108 .7096 .7083 1 2 4 5 6 7<br/>.5 .6 .7 .8 .9 1 2 3 4 5 6</td></td<> | 8 3936 3934 3932 3930 5928 0 0 1 1 1 1 2<br>7 3914 5912 3910 3907 5905 0 0 1 1 1 1 2<br>8 3939 3988 3985 3982 3880 0 1 1 1 1 2   | 5 - 3823 - 3829 - 3828 - 3823 - 3820 0 1 1 1 2 2   | 8 9799 9796 9792 9785 9785 0 1 1 1 2 2 1 1 2 2 1 9763 9755 9751 9748 0 1 1 2 2 2 3 9724 9715 9711 9707 0 1 1 2 2 2  
  | 8. 3681. 3677 3673 3668 3664 0 1 1 2 2 3<br>1. 3638 3632 3627 3622 3617 0 1 1 2 2 3<br>0 5500 5632 3627 3522 3617 0 1 1 2 2 3<br>0 5500 5601 3670 3573 5558 0 1 1 2 2 3  | 2 9537 9532 9527 9521 9516 1 1 2 2 3 3 9 9453 9459 9475 9476 9472 9466 9461 1 1 2 2 3 3 9 9453 9471 9415 9470 9446 9461 1 1 2 2 3 3 3  | 3 9367 9361 9354 9348 9348 9342 1 1 2 2 3 4 4                          | 1.3304.9238.9239.9235.9278         1         1         2         3         4         4           5.3239.9222.9225.9219.9212         1         1         2         3         3         4         6           3.347.9150.9143         1         1         2         3         3         4         6   | 7 .9100 .9022 .9085 .9078 .9070 1 1 2 3 4 4 5<br>3 .9026 .9018 .9011 .9003 .8996 1 2 2 3 4 5 5<br>7 .8949 .8942 .8934 .8926 .8918 1 2 2 3 4 5 5  | 8.8870 8862 8854 8846 8838 1 2 2 3 4 5 6 6 5 9 2 9 2 9 2 3 4 5 6 6 5 9 2 9 2 9 2 1 2 2 3 4 5 6 6 9
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   |
| x         0         ·i         2           7         0         1.000         1.000         -000           7         0.9996         9974         -9973         -           7         0.9996         9974         -9973         -           8         0.9996         9974         -9973         -           7         0.9996         9974         -9973         -           9         0.9996         9974         -         -           9         0.9967         9974         -         -           9         0.9967         9974         -         -           10         0.9945         9943         9974         -           11         0.9946         9913         9919         9774           11         0.9946         9913         9919         9774           11         0.9946         9913         9714         9917           11         0.9945         9443         9713         9919         9918           11         0.9945         9443         9713         9919         9913           11         0.9945         9443         9713         9913         9913 <td>ALL THE PLAT THE PLAT THE PLAT THE PLAT THE PLAT DIffer</td> <td>4 .5 .6 .7 .8 .9 1 2 3 4 5 6</td> <td>6666* 6666* 6666* 6666* 000°L 000°L</td> <td>-9997         -9996         -9997         -9917         -9917         -9917         -9916         <td< td=""><td></td><td>-9938 -9936 -9934 -9932 -9330 -9928 0 0 1 1 1 1 2<br/>-9917 -9914 -9912 -9910 -9907 -9905 0 0 1 1 1 1 2<br/>-9983 -9890 -9898 -9888 -9882 -9880 0 1 1 1 1 2</td><td></td><td>9803 9799 9796 9792 9789 9785 0 1 1 1 2 2 9767 9763 9759 9755 9751 9748 0 1 1 2 2 2 9728 9728 9724 9720 9715 9711 9707 0 1 1 2 2 2</td><td>-9686 -9681 -9677 -9673 -9668 -9664 0 1 1 2 2 3<br/>-9641 -9963 -9632 -9627 -9557 -967 0 1 1 2 2 3<br/>-0641 -9080 -9620 -9672 -967 0 1 1 2 2 3</td><td>9542 9537 9532 9527 9521 9516 1 1 2 2 3 3 9489 9489 9489 9489 9481 9412 9416 9417 9415 9409 9413 1 1 2 2 3 3 3 00000 0410 9412 9416 9400 9410 9412 3 3 3 3 00000 0410 9412 9413 9410 9413 9410 9413 9410 9410 9410 9410 9410 9410 9410 9410</td><td>9373 - 3367 - 9361 - 9354 - 9348 - 9342 - 1 1 2 2 3 4 4</td><td>9311. 9304. 9298 9291. 9285. 9278 1 1 2 3 3 4 4 9151. 9245. 9239. 9285. 9219. 9212 1 1 2 3 3 4 4 9171. 9164. 9157. 9150. 9143 1 1 2 3 3 4 1</td><td>3107 3100 3092 3085 3078 3070 1 1 2 3 4 4 5 5 3 3053 3055 3019 3091 300 3996 1 2 2 3 4 5 5 5 3 4 5 5 1 1 2 2 3 4 5 5 1 1 2 2 3 4 5 5 1 1 2 2 3 4 5 5 1 1 2 2 1 3 4 5 5 1 1 2 2 1 3 4 5 1 1 2 1 2 1 3 4 5 1 1 1 2 1 1 2 1 3 4 5 1 1 1 2 1 1 2 1 3 4 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td><td>8878 8870 8862 8854 8846 8838 1 2 2 3 4 5 6 9 9 1 2 2 3 4 5 0 9 1 2 2 3 4 5 0 9 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 3 3 3 4 5 0 1 2 3 3 3 3 3 4 5 0 1 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3</td><td>8625 8616 8607 8599 8590 8581 1 2 3 4 4 5</td><td>8536         8517         8506         8490         1         2         3         4         5         5         6           8443         8425         8415         8406         8396         1         2         3         4         5         6         7           8443         8425         8415         8406         8396         1         2         3         4         5         6         7           8443         8239         8330         1         2         3         4         5         6         7           8443         8339         8330         1         2         3         4         5         6         7</td><td>-8251         8231         8221         8211         8202         1         2         3         4         5         6         7           8151         8141         8121         8111         8100         1         2         3         4         5         6         7           8151         8141         8121         8111         8100         1         2         3         4         5         6         7           8151         8121         8111         8100         1         2         3         4         5         6         7</td><td></td><td>.712/ .718 .718 .718 .718 .718 .718 .718 .718</td><td>7501         7490         7478         7456         7455         7443         1         2         5         6         7           7335         7337         7339         7337         7325         1         2         4         5         6         7           7355         7333         7361         7339         7332         7325         1         2         4         5         6         7           7266         7242         7230         7231         7205         1         2         4         5         6         7</td><td>-7145 -7133 -7120 -7108 -7096 -7083 1 2 4 5 6 7<br/>.4 5 6 7 .8 .9 1 2 3 4 5 6</td></td<></td>  
  | ALL THE PLAT THE PLAT THE PLAT THE PLAT THE PLAT DIffer                   | 4 .5 .6 .7 .8 .9 1 2 3 4 5 6           | 6666* 6666* 6666* 6666* 000°L 000°L                   | -9997         -9996         -9997         -9917         -9917         -9917         -9916 <td< td=""><td></td><td>-9938 -9936 -9934 -9932 -9330 -9928 0 0 1 1 1 1 2<br/>-9917 -9914 -9912 -9910 -9907 -9905 0 0 1 1 1 1 2<br/>-9983 -9890 -9898 -9888 -9882 -9880 0 1 1 1 1 2</td><td></td><td>9803 9799 9796 9792 9789 9785 0 1 1 1 2 2 9767 9763 9759 9755 9751 9748 0 1 1 2 2 2 9728 9728 9724 9720 9715 9711 9707 0 1 1 2 2 2</td><td>-9686 -9681 -9677 -9673 -9668 -9664 0 1 1 2 2 3<br/>-9641 -9963 -9632 -9627 -9557 -967 0 1 1 2 2 3<br/>-0641 -9080 -9620 -9672 -967 0 1 1 2 2 3</td><td>9542 9537 9532 9527 9521 9516 1 1 2 2 3 3 9489 9489 9489 9489 9481 9412 9416 9417 9415 9409 9413 1 1 2 2 3 3 3 00000 0410 9412 9416 9400 9410 9412 3 3 3 3 00000 0410 9412 9413 9410 9413 9410 9413 9410 9410 9410 9410 9410 9410 9410 9410</td><td>9373 - 3367 - 9361 - 9354 - 9348 - 9342 - 1 1 2 2 3 4 4</td><td>9311. 9304. 9298 9291. 9285. 9278 1 1 2 3 3 4 4 9151. 9245. 9239. 9285. 9219. 9212 1 1 2 3 3 4 4 9171. 9164. 9157. 9150. 9143 1 1 2 3 3 4 1</td><td>3107 3100 3092 3085 3078 3070 1 1 2 3 4 4 5 5 3 3053 3055 3019 3091 300 3996 1 2 2 3 4 5 5 5 3 4 5 5 1 1 2 2 3 4 5 5 1 1 2 2 3 4 5 5 1 1 2 2 3 4 5 5 1 1 2 2 1 3 4 5 5 1 1 2 2 1 3 4 5 1 1 2 1 2 1 3 4 5 1 1 1 2 1 1 2 1 3 4 5 1 1 1 2 1 1 2 1 3 4 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td><td>8878 8870 8862 8854 8846 8838 1 2 2 3 4 5 6 9 9 1 2 2 3 4 5 0 9 1 2 2 3 4 5 0 9 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 2 3 4 5 0 1 2 3 3 3 4 5 0 1 2 3 3 3 3 3 4 5 0 1 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3</td><td>8625 8616 8607 8599 8590 8581 1 2 3 4 4 5</td><td>8536         8517         8506         8490         1         2         3         4         5         5         6           8443         8425         8415         8406         8396         1         2         3         4         5         6         7           8443         8425         8415         8406         8396         1         2         3         4         5         6         7           8443         8239         8330         1         2         3         4         5         6         7           8443         8339         8330         1         2         3         4         5         6         7</td><td>-8251         8231         8221         8211         8202         1         2         3         4         5         6         7           8151         8141         8121         8111         8100         1         2         3         4         5         6         7           8151         8141         8121         8111         8100         1         2         3         4         5         6         7           8151         8121         8111         8100         1         2         3         4         5         6         7</td><td></td><td>.712/ .718 .718 .718 .718 .718 .718 .718 .718</td><td>7501         7490         7478         7456         7455         7443         1         2         5         6         7           7335         7337         7339         7337         7325         1         2         4         5         6         7           7355         7333         7361         7339         7332         7325         1         2         4         5         6         7           7266         7242         7230         7231         7205         1         2         4         5         6         7</td><td>-7145 -7133 -7120 -7108 -7096 -7083 1 2 4 5 6 7<br/>.4 5 6 7 .8 .9 1 2 3 4 5 6</td></td<> |   
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| x         0         1           x         0         1         0000         1           1         0         1         0         1           3         0         0         1         0000         1           4         0         0         1         0000         1         0000           5         0         0         0         1         0000         1         0000           5         0         0         99368         99938         99938         99938         99938         99938         99938         99934         99934         99934         99934         99934         99934         99934         99934         99934         99934         99934         99934         99934         99934         99934         99148         99148         99148         99148         99148         99148         99148         99148         99134  
  | THE PLACE PART AND                    | 3 4 5 6 7 8 9 1 2 3 4 5 6              | 6665 6888 8888 000-L 000-L                            | 9397, 3997, 3992, 3996, 3996, 3995, 3996<br>9992, 3999, 3990, 3999, 3988, 3987, 0 0 0 0 0 0 0 1 1<br>9982, 3982, 3981, 3980, 3979, 3978, 3977, 0 0 0 1 1 1  | 9972 .9971 .9956 .9958 .9958 .9958 .9953 0 0 0 1 1 1 1 9955 .9955 .9954 .9952 .9951 .9947 0 0 1 1 1 1 1   
   | 9940 9938 9936 9944 9922 3930 9928 0 0 1 1 1 1 2 9919 9919 9919 9914 9912 9910 9905 0 0 1 1 1 1 2 8005 9803 9809 9809 0 1 1 1 1 2 2 8005 9809 0 0 1 1 1 1 2  |  | 8806 9803 9799 9796 9792 9789 9785 0 1 1 1 2 2 9770 9767 9763 9755 9755 9748 0 1 1 2 2 2 9722 9722 9722 9715 9711 9707 0 1 1 2 2 2  
  | 9630 3688 3681 3677 3678 3668 9664 0 1 1 2 2 3<br>9646 3641 9638 9653 9573 9525 9517 0 1 1 2 2 3<br>0000 6670 6670 6673 6673 0673 058  | 9548 9542 9537 9532 9527 9521 9516 1 1 2 2 3 3 9349 9489 9483 9483 9418 9472 9486 9461 1 1 2 2 3 3 3   | 9379 9373 -3357 -93581 -9354 -9348 -9342 -1 1 2 2 3 4 4                | 9317 9311 9304 9298 9291 9285 9278 1 1 2 3 3 4 9<br>9262 9245 9239 9225 9219 9212 1 1 2 3 3 4 9<br>9184 9178 9171 9164 9157 9150 9143 1 1 2 3 3 4 9   | 9114 9107 9100 9092 9085 9078 9070 1 1 2 3 4 4 5<br>9041 9023 9028 9018 9011 9003 8996 1 2 2 3 4 5 5<br>9041 9023 8964 1 2 2 3 4 5 5   | 8886 8878 8870 8852 8854 8846 8838 1 2 2 3 4 5 6 8806 8796 978 8790 877 575 8765 1 2 2 3 4 5 6   
   | -9624 -9625 -8616 -8607 -8599 -8590 -8581 -1 2 3 4 4 5                         | 8545         8526         8517         8508         8490         1         2         3         4         5         5         6           8453         8443         8415         8406         8396         1         2         3         4         5         5         6           8453         8446         8406         8396         1         2         3         4         5         6         7           8358         8338         8339         8310         8310         8300         1         2         3         4         5         6         7   | 8261 8251 8241 8231 8231 8231 8202 11 2 3 4 5 6 7 2 8561 8151 814 8131 8121 811 8100 1 2 3 4 5 6 7 7 844 8131 8131 8100 1 2 3 4 5 6 7   |  | -7788 :7721 ·1716 ·7/15 ·7/15 ·764 ·7693 ·7541 ·7570 ·7559 1 2 3 5 6 7   |   | -7157 -7145 -7133 -7120 -7108 -7083 1 2 4 5 6 7<br>  
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| x         0         ···           x         0         1-0000         1-00           1         0         0-9995         999           3         0-99955         999         999           4         0-99976         999         999           7         0-99975         999         999           9         0-99976         999         999           9         0-99976         999         999           9         0-99975         999         999           9         0-99975         999         991           9         0-99975         991         991           111         0-99975         991         991           111         0-99455         944         941           111         0-99455         944         941           111         0-99455         944         941           111         0-94455         944         941           111         0-9455         944         941           111         0-9455         944         941           111         0-9455         945         942           111         0-9455         94  
  | LE BUSO LAT BUSO ALE CON ALE CALL CONT A CONTRACT Differ                  | 2 .3 .4 .5 .6 .7 .8 .9 1 2 3 4 5 6     | 000 1-000 1-000 -3939 -3939 -3939 -3939               | 908 .0997 .9997 .9997 .9997 .9996 .9996 .9995 .9995 .9995 .9995 .9995 .9997 .0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1  | 773 .9972 .9971 .9963 .9968 .9966 .9965 .9963 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1   
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 7096         7096 |
| x         0         1,0000           x         0         1,0000           1         0,09998         3           2         0,99996         3           3         0,99956         5           4         0,99976         5           5         0,99956         5           6         0,99456         5           7         0,99268         5           9         0,99266         5           11         0,99316         5           11         0,99316         5           12         0,99263         5           13         0,94563         5           14         0,99316         5           15         0,94553         5           16         0,94553         5           17         0,99533         5           18         0,9135         5           222         0,9333         5           233         0,98572         23           333         0,8487         33           333         0,8487         33           333         0,98573         38           0,01,97333         0,77344 <td>CALL BLOOD JAYS BLADE BLADE BLADE CALL TOOLS IN SUBTRACT Differ</td> <td>2 3 4 5 6 7 8 9 1 2 3 4 5 6</td> <td>0 1-000 1-000 1-000 -3939 -3939 -3939 -3939</td> <td>8. 3938. 3997 3937. 3937. 3936. 3936. 3936. 3936. 3935. 3938<br/>3. 39932. 3932. 39391. 3930. 3930. 3939. 3938. 3937<br/>5. 3934. 3932. 3932. 3930. 3930. 3973. 3977. 0 0 0 0 1 1 1</td> <td>4 .973 .9972 .9971 .9965 .9968 .9966 .9965 .9963 0 0 0 1 1 1 1 0 0 .9955 .9957 .9954 .9952 .9951</td> <td>3 3942 3940 3938 3936 3934 3932 3930 5952 0 0 1 1 1 1 2<br/>3 3921 3919 3917 3914 3912 3910 3907 3905 0 0 1 1 1 1 2<br/>0 3888 3895 3893 3893 3888 9855 5882 3880 0 1 1 1 2</td> <td>4 -36/1 -3606 -3603 -3603 -3604 -3644 -3641 - 5 - 4 -36/1 - 5 - 3833 -3833 -3823 -38</td> <td>3 3910 3906 3603 3799 5756 3792 3789 3785 0 1 1 1 2 2 8 3774 3770 3767 3753 3759 3751 3748 0 1 1 2 2 2 0 3736 3732 3732 3751 3771 3707 0 1 1 2 2 2</td> <td>9. 9694. 9680. 9681. 9677 9673. 9668. 9664         0         1         1         2         2           5. 9660. 9646. 9641. 9653. 9652. 9657. 9657         0         1         1         2         3           6. 9660. 9646. 9641. 9653. 9652. 9657         0         1         1         2         3</td> <td>06         300.0         430.0         530.0         530.0         530.0         530.0         530.0         553.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         552.0         543.0         54</td> <td>11 9385 9379 9377 9387 9381 9384 9348 9342 1 1 2 2 3 4 4</td> <td>00.3323.3317 3311.3304.9288.3291.3285.3278 1 1 2 3 3 4 4 5 5 5 2559.9228 3239 3225 3219 3212 1 1 2 3 3 4 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5</td> <td>28. 3-121         3-114         3-107         3-100         3-025         3-065         3-078         3-070         1         1         2         3         4         5         5         5         5-050         3-07</td> <td>22 8894 8886 8879 8870 8852 8854 8846 8838 1 2 2 3 4 5 6 1 1 8813 8805 8795 8789 878 8846 8838 1 2 2 3 4 5 6 6 1 0 8813 8805 8795 8785 8785 8785 8785 8785 878 846 1 2 3 3 4 5 6</td> <td>22. 2643. 3634 3625. 9616. 9607 3599 3590 3551 1 2 3 4 4 5</td> <td>33. 8554. 8545         8526. 8526         8517. 8508. 9499         1         2         3         4         5         5         6           71. 82452         8453         8443         8425         8415         9406         3396         1         2         3         4         5         5         6           77. 8265         8358         8348         8329         8320         8310         3310         3         4         5         6         7           77. 8368         8358         8348         8329         8320         8310         8310         3300         1         2         3         4         5         6         7</td> <td>81 8271 8261 8251 8241 8231 8231 8231 8202 11 2 3 4 5 6 7<br/>31 3171 8161 8151 8141 8141 8113 8121 8113 8100 1 2 3 4 5 6 7<br/>2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2</td> <td>00 - 80/0 - 80/05 - 7955 - 7924 - 7924 - 7924 - 7912 - 7902 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 -
7793 - 7792 - 7792 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 7793 - 779</td> <td>00 - 7/49 - 7738 - 7727 - 7/16 - 7/10 - 7/094 - 7063 - 7072 - 1 2 3 7 9 9 7 9 9 - 7 627 - 7 627 - 7 664 - 7 593 - 7 581 - 7 570 - 7 559 - 1 2 3 5 6 7</td> <td>36 7524 7513 7501 7490 7478 7466 7455 7443 1 2 3 5 6 7<br/>20 7408 7396 7395 7373 7351 7349 7337 7325 1 2 4 5 6 7<br/>22 7290 77278 7266 7254 7724 7720 7218 7206 1 2 4 5 6 7</td> <td>81 .7169 .7157 .7145 .7133 .7120 .7108 .7096 .7095 .7095 .7053 1 2 4 5 6 7<br/>.2 .3 .4 .5 .6 .7 .8 .9 1 2 3 4 5 6</td> | CALL BLOOD JAYS BLADE BLADE BLADE CALL TOOLS IN SUBTRACT Differ           | 2 3 4 5 6 7 8 9 1 2 3 4 5 6            | 0 1-000 1-000 1-000 -3939 -3939 -3939 -3939           | 8. 3938. 3997 3937. 3937. 3936. 3936. 3936. 3936. 3935. 3938<br>3. 39932. 3932. 39391. 3930. 3930. 3939. 3938. 3937<br>5. 3934. 3932. 3932. 3930. 3930. 3973. 3977. 0 0 0 0 1 1 1   | 4 .973 .9972 .9971 .9965 .9968 .9966 .9965 .9963 0 0 0 1 1 1 1 0 0 .9955 .9957 .9954 .9952 .9951   
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  | 1.9639         9680         9688         9681         9677         9673         3668         9684         0         1         1         2         2         3           9655         9560         9641         9631         9573         9523         9517         1         1         2         2         3           0         0         1         1         2         2         3 | 1.9006         9003         9500         9503         9532         9531         9516         1         1         2         3         9505         9509         9499         9493         9432         9438         9431         9415         1         1         2         3         3         9           9505         9509         9499         9432         9435         9445         9445 <td>9 3391 3385 3379 3373 3387 3381 3354 3348 3348 3342 1 1 2 2 3 4 4</td> <td>6         3333         3317         3314         3304         9298         9285         9218         9218         1         1         2         3         4         4           1         9265         9269         9285         9219         9212         1         1         2         3         3         4         5           1         9265         9269         9285         9232         9223         9223         9223         9212         1         1         2         3         3         4         5           9198         9191         9184         9178         9171         9164         9157         9150         9143         1         2         3         4         1</td> <td>1, 3128         -3121         -3100         -3032         -3085         -3073         -3070         1         1         2         3         4         5           1         3055         -9048         -9011         -9012         -9003         -9961         1         2         3         4         5         5           1         3055         -9018         -9011         -9012         -9003         -9961         1         2         3         4         5         5           1         4055         -8048         -8041         -8042         -8044         -8043         -8044         -804</td> <td>0.000 000 000 000 000 000 000 000 000 0</td> <td>9 00/30 00/22 00/21 00/12 00/04 00/00 00/00 00/0 00/02 00 00/0</td> <td>2         8563         8554         8556         8517         8508         8499         7         2         3         4         5         5         6           1         8471         8462         8543         8445         8415         8416         8396         1         2         3         4         5         6         7           1         8471         8462         8418         8306         1         2         3         4         5         6         7           1         8471         8462         8415         8406         8396         1         2         3         4         5         6         7           1         8337         8338         8339         8339         8320         8310         810         2         3         4         5         6         7</td> <td>0 8281 8271 8261 8251 8251 8231 8231 821 820 11 2 3 4 5 6 7 8 8 8 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9</td> <td></td> <td>1 1,7760 -7749 -7738 -7721 -7716 -7709 -7094 -7054 -7054 -7520 -7559 -7523 -7521 -7570 -7559 -7527 -7515 -7504 -7533 -7551 -7570 -7559 -7570 -7559 -7529 -7759 -7529 -75</td> <td>7. 7556. 7524. 7513. 7501. 7490. 7478. 7466. 7455. 7443. 11 2. 3 5 6 7<br/>1. 7420. 7408. 7396. 7385. 7373. 7361. 7349. 7337. 7325. 11 2. 4 5 6 7<br/>1. 7302. 7290. 7278. 7268. 7724. 7242. 7230. 7218. 7206. 11 2. 4 5 6 7</td> <td>3 .7181 .7163 .7157 .7145 .7133 .7120 .7108 .7096 .7083 1 2 4 5 6 7<br/>.1</td> | 9 3391 3385 3379 3373 3387 3381 3354 3348 3348 3342 1 1 2 2 3 4 4      | 6         3333         3317         3314         3304         9298         9285         9218         9218         1         1         2         3         4         4           1         9265         9269         9285         9219         9212         1         1         2         3         3         4         5           1         9265         9269         9285         9232         9223         9223         9223         9212         1         1         2         3         3         4         5           9198         9191         9184         9178         9171         9164         9157         9150         9143         1         2         3         4         1 | 1, 3128         -3121         -3100         -3032         -3085         -3073         -3070         1         1         2         3         4         5           1         3055         -9048         -9011         -9012         -9003         -9961         1         2         3         4         5         5           1         3055         -9018         -9011         -9012         -9003         -9961         1         2         3         4         5         5           1         4055         -8048         -8041         -8042         -8044         -8043         -8044         -804  | 0.000 000 000 000 000 000 000 000 000 0  
   | 9 00/30 00/22 00/21 00/12 00/04 00/00 00/00 00/0 00/02 00 00/0                 | 2         8563         8554         8556         8517         8508         8499         7         2         3         4         5         5         6           1         8471         8462         8543         8445         8415         8416         8396         1         2         3         4         5         6         7           1         8471         8462         8418         8306         1         2         3         4         5         6         7           1         8471         8462         8415         8406         8396         1         2         3         4         5         6         7           1         8337         8338         8339         8339         8320         8310         810         2         3         4         5         6         7   | 0 8281 8271 8261 8251 8251 8231 8231 821 820 11 2 3 4 5 6 7 8 8 8 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9   |  | 1 1,7760 -7749 -7738 -7721 -7716 -7709 -7094 -7054 -7054 -7520 -7559 -7523 -7521 -7570 -7559 -7527 -7515 -7504 -7533 -7551 -7570 -7559 -7570 -7559 -7529 -7759 -7529 -75 | 7. 7556. 7524. 7513. 7501. 7490. 7478. 7466. 7455. 7443. 11 2. 3 5 6 7<br>1. 7420. 7408. 7396. 7385. 7373. 7361. 7349. 7337. 7325. 11 2. 4 5 6 7<br>1. 7302. 7290. 7278. 7268. 7724. 7242. 7230. 7218. 7206. 11 2. 4 5 6 7  | 3 .7181 .7163 .7157 .7145 .7133 .7120 .7108 .7096 .7083 1 2 4 5 6 7<br>.1  
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  | REAU DIG MAY REAR REAR BAL TORS FOR THE REAL TORS FOR THE SUBTRACT DIffer | 0 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6        | 1-0000 1-000 1-000 1-000 1-000 -3939 -3939 -3939      | 0.3338 3938 3938 3937 3937 3937 3934 3936 3936 3935 3936 3936 3936 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0  | 0.3376 .3974 .3973 .3972 .3977 .3958 .3958 .3958 .3956 .3953 0 0 0 1 1 1 1 0 0.9562 .3956 .3954 .3957 .3954 .3954 .3954 .3954 .3954 .3954 .3957 .3954 .3957 .   
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Cosines of Angles (x in degrees)

# Appendix IV: Tangents of Angles

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x         d           x         d           0         0.0000           1         0.0175           2         0.0275           6         0.0175           7         0.1228           9         0.0575           6         0.0175           9         0.0275           6         0.1228           9         0.1228           9         0.1228           13         0.1228           13         0.1228           13         0.2209           14         0.2309           15         0.2128           16         0.2128           17         0.2209           16         0.2309           17         0.2309           18         0.2309           19         0.2445           22         0.4462           23         0.4462           23         0.4462           23         0.5343           23         0.5445           23         0.5544           23         0.5543           24         0.5544           33         0.5574 <td></td> <td>÷</td> <td>0017</td> <td>0192</td> <td>2120</td> <td>0692</td> <td>1245</td> <td>1602 -</td> <td>1781</td> <td>2144</td> <td>2512</td> <td>3076</td> <td>3659</td> <td>4061 4265</td> <td>4473</td> <td>5117</td> <td>5797</td> <td>6032 6273 6519</td> <td>1773</td> <td>7563</td> <td>8127</td> <td>8724 5036 5036</td> <td>1606</td> <td>-</td> <td></td>		÷	0017	0192	2120	0692	1245	1602 -	1781	2144	2512	3076	3659	4061 4265	4473	5117	5797	6032 6273 6519	1773	7563	8127	8724 5036 5036	1606	-	
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Tangents of Angles (x in degrees)

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Appendix V: Square Roots of Numbers, 1-10

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Appendix VI: Square Roots of Numbers, 10-100

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  4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819         4.893         4.819 <td< td=""><td>20 4-540 4-560 4-570 4-590 4-590 1 2 3 4 5 6 7 8 9 10 5-040 5-050 5-050 5-075 5-079 1 2 3 4 5 6 7 8 9 18 5 1548 5-145 5-175 5-109 1 2 3 4 5 6 7 8 9</td><td>25         5.235         5.244         5.263         5.213         5.202         1         2         3         4         5         6         7         8         9           10         5.235         5.244         5.263         5.213         5.202         1         2         3         4         5         6         7         8         9           15         5.425         5.431         5.541         5.537         1         2         3         4         5         6         7         8           15         5.422         5.431         5.415         5.441</td><td>05 5514 5523 5541 5550 5559 1 2 3 4 4 5 6 7 8</td><td>96 5604 5612 5621 5630 5639 5648 1 2 3 3 4 5 6 7 8 33 5632 5701 5770 5771 5778 5725 1 2 3 3 4 5 6 7 8 11 5779 5788 5797 5805 5614 5822 1 2 3 3 4 5 6 7 8</td><td>77 5.855 5.874 5.822 5.891 5.839 5.908 1 2 2 3 3 4 5 6 7 8 11 5565 5-958 5-957 5-953 5-932 1 2 2 3 4 4 5 6 7 8 15 6.033 6.402 6.056 6.507 6 6.075 1 2 2 3 4 4 5 6 7 7</td><td>7) 6116 6124 6122 6140 6148 6156 1 2 2 3 4 5 6 7 7 29 6197 6205 6213 6229 6223 6231 1 2 2 3 4 5 6 7 7 19 6277 6226 5231 6301 6309 6317 1 2 2 3 4 5 6 6 7</td><td>1 6 356 6 364 6 372 6 380 6 387 6 395 1 2 2 3 4 5 6 6 7</td><td>77         6-434         6-462         6-458         6-465         6-453         6-453         6-57         6-513         6-57         6-513         6-57         6-513         6-513         6-52         6-513         6-513         6-51         6-51         6-51         6-51         6-51         6-51         6-51         6-52         6-51         6-52         6-51         6-52         6-51         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-52         6-7         7         9         6-58         6-52         6-7         7         9         6-58         6-7         7         9         6-58         6-7         7         9         6-58         6-7         7         9         6-58         6-7         7         9         6-58         6-7         7         9         6-58         6-7         7         9         6-7         7         9         6-7         7         9         6-7         7         9         6-7         7         9         6-7         7         9         6-7         7         9         <td< td=""><td>56 6663 6671 6678 6686 6633 6701 1 2 2 3 4 5 5 6 7 11 6738 6745 6733 5706 6788 6775 1 1 2 3 4 4 5 5 7 14 6615 6619 66306 5534 6841 68-41</td><td>8         6.835         6.897         6.907         6.914         6.921         1         1         2         3         4         5         6         7           10         56.855         6.897         6.907         6.914         6.921         1         1         2         3         4         5         6         7           10         56.95         6.897         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.997         6.994         1         2         3         4         5         6         6         6         7.944         1         2         3         4         5         6         6         7.943         7.944         1         2         3         4         5         6         6         6         7.943</td><td>12 7-099 7-106 7-113 7-120 7-127 7-134 1 1 2 3 4 4 5 6 6</td><td>22 7.169 7.176 7.182 7.190 7.197 7.204 1 1 2 3 4 4 5 6 6 7 7 7.239 7.246 7.253 7.266 7.273 1 1 2 3 3 4 5 6 6 1 1 7.308 7.314 7.321 7.328 7.335 7.342 1 1 2 3 3 4 5 5 6</td><td>39 7.376 7.382 7.389 7.396 7.403 7.410 1 1 2 3 3 4 5 5 6</td></td<></td></td<>	20 4-540 4-560 4-570 4-590 4-590 1 2 3 4 5 6 7 8 9 10 5-040 5-050 5-050 5-075 5-079 1 2 3 4 5 6 7 8 9 18 5 1548 5-145 5-175 5-109 1 2 3 4 5 6 7 8 9	25         5.235         5.244         5.263         5.213         5.202         1         2         3         4         5         6         7         8         9           10         5.235         5.244         5.263         5.213         5.202         1         2         3         4         5         6         7         8         9           15         5.425         5.431         5.541         5.537         1         2         3         4         5         6         7         8           15         5.422         5.431         5.415         5.441	05 5514 5523 5541 5550 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 6         7           10         56.855         6.897         6.907         6.914         6.921         1         1         2         3         4         5         6         7           10         56.95         6.897         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.996         6.997         6.994         1         2         3         4         5         6         6         6         7.944         1         2         3         4         5         6         6         7.943         7.944         1         2         3         4         5         6         6         6         7.943</td><td>12 7-099 7-106 7-113 7-120 7-127 7-134 1 1 2 3 4 4 5 6 6</td><td>22 7.169 7.176 7.182 7.190 7.197 7.204 1 1 2 3 4 4 5 6 6 7 7 7.239 7.246 7.253 7.266 7.273 1 1 2 3 3 4 5 6 6 1 1 7.308 7.314 7.321 7.328 7.335 7.342 1 1 2 3 3 4 5 5 6</td><td>39 7.376 7.382 7.389 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# Square Roots of Numbers 10-100

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5-477 5-568 5-657 5-657 5-745

88833 9884 3373 **8** 53853 5855 53333

6-083 6-164 6-245

5-831 5-916 6-000 6-403 6-481 6-557

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6-633 6-708 6-782 6.856 6.928 7.071 7.141 7.211 7.211 7.210 7.249 7.349

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6-325

# Appendix VII: Reciprocals of Numbers

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×         0         1         2         3         4           11         100         9901         9801         9615         9		6 7 8 9 1 2 3	9434 9346 9259 9174 9 18 28 37 4	8621         8547         8475         8403         8         15         23         31         38           7337         7874         7813         7752         7         13         20         26         33           7353         7299         7246         7194         6         11         17         22         28	8849         6803         6757         6711         5         10         14         19         24         2           6410         6369         6329         6289         4         8         13         17         21         2         2         2         2         12         12         12         2         2         2         2         2         12         12         12         12         12         2         12         12         2         12         2         12         12         12         12         12         12         2         12         12         14         15         18         2         14         14         14         14         14         14         14         14         14         14         14         14         14         15         18         2         15         14         13         15         14         13         17         15         18         2         14         14         14         14         14         14         14         14         14         14         14         14         14         14         14         13         15         15         15         15	682         5650         5618         5537         3         7         10         13         12         12           5376         5348         5319         5291         3         7         10         13         12         10           5376         5348         5319         5291         3         5         8         11         13         15         15         17         17         17         16         17         16         17         16         17         16         17         16         13         5         8         11         13         16         17         16         16         16         16         16         16         17         16         17         16         17         17         17         17         17         17         17         17         17         17         17         17         16	4854 4831 4808 4785 2 5 7 10 12 14	4530 4608 4587 4565 2 4 7 9 11 13 4425 4405 4367 2 4 6 8 10 12 4327 4219 4202 4184 2 4 5 7 9 11	4065 4049 4032 4016 2 3 5 7 8 10 3906 3891 3876 3861 2 3 5 6 8 9 3759 3745 3731 3717 1 3 4 6 7 9	852 3610 3697 3584 1 2 4 5 7 8 3873 3868 377 3584 1 2 4 5 7 8 3378 3567 3364 1 2 3 5 6 7	3268 3257 3247 3236 1 2 3 4 5 6	3165 3155 3145 3135 1 2 3 4 5 6 3067 3058 3049 3040 1 2 3 4 5 5 2976 2967 2959 2950 1 2 3 4 4 5	2880         2882         2874         2865         1         2         3         4         5         6           2809         2801         2793         2786         1         2         3         4         5         6         6           2819         2801         2793         2786         1         2         3         4         5         6         7           2732         2775         2717         2710         1         2         2         3         4         5         5         5	0600         2653         2646         2639         1         1         2         3         4         1           0591         2584         2577         2571         1         1         2         3         4         1           0582         2513         2506         1         1         2         3         3         4	2463 2457 2451 2445 1 1 2 2 3 4 4	2404 2398 2392 2337 1 1 2 2 3 3 234 2268 2283 2278 1 1 2 2 3 3 2294 2268 2283 2278 1 1 2 2 3 3	2242 2237 2232 2227 1. 1 2 2 3 3 193 2188 2183 2179 0 1 1 2 2 3 2146 2141 2137 2132 0 1 1 2 2 3	101         2095         2088         0         1         1         2         3         3           0058         2053         2049         2045         0         1         1         2         2         3         3           0058         2053         2049         2045         0         1         1         2         2         3         3           0016         2012         2008         2004         0         1         1         2         2         3         3	1976 1972 1969 1965 0 1 1 2 2 2	1938         1934         1931         1927         0         1         1         2         2           1901         11898         1894         1890         0         1         1         1         2         2           1866         1862         1855         0         1         1         1         2         2	1828         1825         1821         0         1         1         2         2         2	6 7 8 9 1 2 3 4 5 6
x         0         1         2         3           10         1000         9901         9804         9709         5           11         9091         9008         8929         8850         5           12         5333         5264         5170         5130         5           13         5655         5535         5495         6544         5         5           13         5656         5535         5495         6544         5		5 6 7 8 9 1 2 3	9524 9434 9346 9259 9174 9 18 28 37 4	8696 8621 8547 8475 8403 8 15 23 31 38 8000 7337 7874 7813 7752 7 13 20 26 33 7407 7353 7299 7246 7194 6 11 17 22 28	5897         6849         6803         6757         6711         5         10         14         19         24         2           \$455         6410         6369         6329         6289         4         8         13         17         21         2         2         2         2         12         12         12         2         15         16         10         16         14         14         14         14         14         14         14         14         15         18         2         10         16         16         2         12         15         2         15         12         13         2         14         13         17         16         14         14         14         14         14         14         14         14         14         14         14         14         14         14         14         15         15         15         15         <	5714 5682 5650 5618 5587 3 7 10 13 16 20 3605 5376 5348 5319 5291 3 6 9 12 15 17 3128 5102 5076 5051 5025 3 5 8 11 13 16	<b>1878 4854 4831 4808 4785 2 5</b> 7 <b>10 12 14</b>	8651 4530 4508 4587 4566 2 4 7 9 11 13 444 4425 4405 4367 2 4 6 8 10 12 1255 4237 4219 4202 4184 2 4 5 7 9 11	1082 4065 4049 4032 4016 2 3 5 7 8 10 1922 3906 3891 3876 3861 2 3 5 6 8 9 1774 3759 3745 3731 3717 1 3 4 6 7 9	3836         3823         3810         3597         3584         1         3         4         5         7         8           5036         3823         3840         372         3460         1         3         4         5         7         8           1390         3378         3367         3366         1         2         4         5         6         7           1390         3378         3565         3344         1         2         3         5         6         7	1279 3268 3257 3247 3236 1 2 3 4 5 6	1175 3165 3155 3145 3135 1 2 3 4 5 6 0077 3067 3058 3049 3040 1 2 3 4 5 5 9986 2976 2969 2950 1 2 3 4 4 5	8899         2880         2882         2874         2865         1         2         3         4         5         6           817         2809         2801         2793         2786         1         2         3         4         5         6           740         2732         2725         2717         2710         1         2         3         4         5         5         6	1667         2660         2653         2646         2653         2646         2639         1         1         2         3         4	1469 2463 2457 2451 2445 1 1 2 2 3 4 4	4410 2404 2398 2392 2387 1 1 2 2 3 3 2353 2347 2342 2336 2331 1 1 2 2 3 3 239 2294 2288 2283 2278 1 1 2 2 3 3	2247 2242 2237 2232 2227 1 1 2 2 3 3 198 2193 2188 2183 2179 0 1 1 2 2 3 151 2146 2141 2137 2132 0 1 1 2 2 3	105         2101         2095         2092         2088         0         1         1         2         2         3	980 1976 1972 1969 1965 0 1 1 2 2 2	941 1938 1934 1931 1927 0 1 1 2 2 2 905 1901 1888 1894 1890 0 1 1 1 2 2 889 1866 1862 1859 1855 0 1 1 1 2 2	835 1832 1828 1825 1821 0 1 1 1 2 2 2	5 6 7 8 9 1 2 3 4 5 6
x         0         1         2           11         1000         9901         9804         97           11         9091         9009         9804         97           11         9091         9009         9804         97           12         5535         5536         5595         5535           15         5555         5535         5595         5535           19         5555         5535         5595         5535         5595           19         5555         5535         5536         5495         5           19         5555         5535         5595         5398         3983         3383         3333         3322         3311         3333         3322         3311         3333         3323         3311         3333         3323         3311         3333         3323		4 5 6 7 8 9 1 2 3	1615 9524 9434 9346 9259 9174 9 18 28 37 4	3/72' 8696 8621 8547 8475 8403 8 15 23 31 38 3065 8000 7337 7874 7813 7752 7 13 20 26 33 463 7407 7353 7299 7246 7194 6 11 17 22 28	3944         6897         8849         6803         6757         6711         5         10         14         19         24         2           4494         6452         6410         6359         6329         6329         6389         4         8         13         17         21         2           0368         6061         6024         5388         5952         5917         4         7         11         15         18         2	747 5714 5682 5550 5518 5597 3 7 10 13 15 20 1355 5405 5376 5348 5319 5291 3 6 9 12 15 17 155 5128 5102 5076 5091 5025 3 5 8 11 13 16	902 4378 4854 4831 4808 4785 2 5 7 10 12 14	1673 4651 4630 4608 4587 4566 2 4 7 9 11 13 464 4444 4425 4305 4386 4387 2 4 6 8 10 12 274 4255 4237 4219 4202 4184 2 4 5 7 9 11	0.098 4082 4065 4049 4032 4016 2 3 5 7 8 10 937 3922 3906 3891 3876 3861 2 3 5 6 8 9 788 3774 3759 3745 3731 3717 1 3 4 6 7 9	650         3636         3523         3517         3597         3594         1         3         4         5         7         8           650         3636         3523         3517         3594         1         3         4         5         7         8           650         3639         3497         3484         1         2         4         5         7         8           401         3390         3378         3386         3344         1         2         3         6         7	289 3279 3268 3257 3247 3236 1 2 3 4 5 6	1185 3175 3165 3155 3145 3135 1 2 3 4 5 6 086 3077 3067 3068 3049 3040 1 2 3 4 5 5 994 2985 2976 2967 2959 2950 1 2 3 4 4 5	907 2899 2890 2882 2874 2865 1 2 3 3 4 5 6 7 855 857 809 2809 2793 2786 1 2 2 3 4 5 7 7 1 2 2 2 4 5 5 1 2 2 2 3 4 5 5 1 2 2 2 3 4 5 5 1 2 2 2 3 4 5 5 1 2 2 2 3 4 5 5 1 2 2 2 3 4 5 5 1 2 2 2 3 4 5 1 2 2 3 4 5 1 2 2 2 3 4 5 1 2 3 4 5 1 2 3 3 4 5 1 2 3 4 5 1 2 3 3 4 5 1 1 3 1 3 1 3 1 3 1 3 1 3 1 3 1 3 1 3	674 2667 2660 2653 2646 2639 1 1 2 3 4 4 4 6 6 6 259 2591 2587 2577 2571 1 1 2 3 3 4 5 538 2532 2525 2519 2513 2506 1 1 2 3 3 3 4	475 2469 2463 2457 2451 2445 1 1 2 2 3 4 4	415 2410 2404 2398 2392 2387 1 1 2 2 3 3 358 2353 2347 2342 2336 2331 1 1 2 2 3 3 304 2299 2294 2268 2283 2278 1 1 2 2 3 3	255 2247 2242 2237 2232 2227 1. 1 2 2 3 3 203 2198 2193 2188 2183 2137 0 1 1 2 2 3 156 2151 2146 2141 2137 2132 0 1 1 2 2 3	110         2105         2101         2096         2092         2088         0         1         1         2         2         3	984 1980 1976 1972 1969 1965 0 1 1 2 2 2	946         1941         1934         1931         1927         0         1         1         2         2           908         1905         1901         1898         1894         1893         1894         1937         0         1         1         2         2           908         1905         1901         1898         1894         1856         0         1         1         1         2         2           873         1869         1865         1865         1865         1866         1         1         1         2         2	838 1835 1832 1828 1825 1821 0 1 1 1 2 2 2	4 5 6 7 8 9 1 2 3 4 5 6
x         0         1           10         1000         9901         980           11         9091         9003         8901           12         8233         8264         81           13         8255         5555         5555           14         6657         6251         65255           15         6250         6211         61           15         6256         5535         534         41           15         6256         5535         533         834         41           16         6250         6231         633         633         63         64           20         5000         456         433         4143         414         41         414 </td <td></td> <td>3 4 5 6 7 8 9 1 2 3</td> <td>1709 9615 9524 9434 9346 9259 9174 9 18 28 37 4</td> <td>8850         8772'         88547         8475         8403         8         15         23         31         38           1130         8065         8000         7937         7874         7813         7752         7         13         20         23         31         38           1519         7463         7407         7353         7299         7246         7194         6         11         17         22         28</td> <td>8033 8044 8697 8949 8803 6757 6711 5 10 14 19 24 2 1536 6494 4552 610 5595 6529 2589 4 8 13 17 21 2 1535 6098 6051 6024 5588 5552 917 4 7 11 15 18 2</td> <td>780         5714         5682         5650         5618         5557         3         7         10         13         16         20           484         5435         5405         53248         53319         5231         3         7         10         13         15         20           484         5435         5405         53248         53319         5231         3         7         10         13         16         10           484         1315         5128         5102         5076         5075         3         5         8         11         3         5         8         11         13         15         502         3         5         8         11         3         5         8         11         3         5         3         3         8         11         3         5         3         3         5         3</td> <td>926 9902 4378 4854 4831 4808 4785 2 5 7 10 12 14</td> <td>865 4673 4551 4530 4568 4567 4566 2 4 7 9 11 13 484 4464 4425 4405 4366 4367 2 4 6 8 10 12 222 4274 4255 4237 4219 4202 4184 2 4 5 7 9 11</td> <td>1115 4098 4082 4065 4049 4032 4016 2 3 5 7 8 10 953 3337 3322 3906 3891 3876 3861 2 3 5 6 8 9 802 3748 3774 3759 3745 3751 2 3 4 6 7 9</td> <td>883 3650 3636 3838 3810 3597 3584 1 3 4 5 7 8 534 3521 3509 3497 3484 3472 3460 1 2 4 5 6 7 1413 2401 3390 3378 3367 3356 3344 1 2 3 5 6 7</td> <td>300 3289 3279 3268 3257 3247 3236 1 2 3 4 5 6</td> <td>115 3125 3175 3165 3155 3145 3135 1 2 3 4 5 6 096 3088 3077 3067 3058 3049 3040 1 2 3 4 5 5 003 2994 2995 2976 2967 2959 2950 1 2 3 4 4 5</td> <td>915 2907 2899 2890 2882 2874 2865 1 2 3 3 4 5 6 8 8 83 2825 2817 2809 2801 2793 2786 1 2 2 3 4 5 6 755 755 2717 2710 1 2 2 3 4 5 5</td> <td>681         26574         2667         26653         2646         2653         2646         2653         1         1         2         3         4         4         4           6111         2604         2591         2564         2577         2571         1         1         2         3         4         4         4           545         2554         2577         2571         2513         2566         1         1         2         3         4         4         4</td> <td>481 2475 2469 2463 2457 2451 2445 1 1 2 2 3 4 4</td> <td>4211 2415 2410 2404 2338 2392 2387 1 1 2 2 3 3 364 2358 2353 2347 2342 2336 2331 1 1 2 2 3 3 3 309 2304 2299 2294 2288 2283 2278 1 1 2 2 3 3 3</td> <td>257 2252 2247 2242 2237 2232 2227 1 1 2 2 3 3 208 2203 2203 208 2192 113 2137 2132 0 1 1 2 2 3 100 1156 2151 2146 2141 2137 2132 0 1 1 2 2 2 3</td> <td>114         2105         2101         2055         2092         2088         0         1         1         2         2         3         3           070  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    23         31         38           1130         8065         8000         7937         7874         7813         7752         7         13         20         23         31         38           1519         7463         7407         7353         7299         7246         7194         6         11         17         22         28	8033 8044 8697 8949 8803 6757 6711 5 10 14 19 24 2 1536 6494 4552 610 5595 6529 2589 4 8 13 17 21 2 1535 6098 6051 6024 5588 5552 917 4 7 11 15 18 2	780         5714         5682         5650         5618         5557         3         7         10         13         16         20           484         5435         5405         53248         53319         5231         3         7         10         13         15         20           484         5435         5405         53248         53319         5231         3         7         10         13         16         10           484         1315         5128         5102         5076         5075         3       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83 2825 2817 2809 2801 2793 2786 1 2 2 3 4 5 6 755 755 2717 2710 1 2 2 3 4 5 5	681         26574         2667         26653         2646         2653         2646         2653         1         1         2         3         4         4         4           6111         2604         2591         2564         2577         2571         1         1         2         3         4         4         4           545         2554         2577         2571         2513         2566         1         1         2         3         4         4         4	481 2475 2469 2463 2457 2451 2445 1 1 2 2 3 4 4	4211 2415 2410 2404 2338 2392 2387 1 1 2 2 3 3 364 2358 2353 2347 2342 2336 2331 1 1 2 2 3 3 3 309 2304 2299 2294 2288 2283 2278 1 1 2 2 3 3 3	257 2252 2247 2242 2237 2232 2227 1 1 2 2 3 3 208 2203 2203 208 2192 113 2137 2132 0 1 1 2 2 3 100 1156 2151 2146 2141 2137 2132 0 1 1 2 2 2 3	114         2105         2101         2055         2092         2088         0         1         1         2         2    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9259 9174 9 18 28 37 4	29         8850         8772         8696         8621         8547         8475         8403         8         15         23         31         38           97         8130         8065         8000         7937         7874         7813         7752         7         13         20         23         31         38           76         7130         8065         8000         7937         7874         7813         7752         7         13         20         26         33           76         7519         7467         7333         7299         7246         7194         6         11         17         22         28	42 8093 8044 8897 8849 8803 6757 6711 5 10 14 19 24 2 79 6530 8049 6452 6410 6589 5322 6589 4 8 73 17 21 2 73 6155 6098 6161 6024 5588 5952 5917 4 7 11 15 18 2	14         5780         5747         5714         5682         5560         5518         5557         3         7         10         13         16         20           14         5780         5747  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 1         2         3         4         5         6           12         3003         2994         2996         2959         2950         1         2         3         4         5         6	24 2915 2007 2899 2890 2882 2874 2865 1 2 3 3 4 5 6 4 1 2823 2825 2817 3809 2801 2932 296 1 2 2 3 4 5 5 8 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	88         2681         2574         2660         2653         2646         2630         1         1         2         3         4         4           18         2611         2604         2591         2584         2577         2571         1         1         2         3         4         4           18         2611         2604         2591         2584         2577         2571         1         1         2         3         4         4           18         2615         2532         25547         2571         2571         1         1         2         3         4         4           18         2615         2538         2557         25572         2552         2552         2552         2553         2556         1         1         2         3         4         4	38 2481 2475 2469 2463 2457 2451 2445 1 1 2 2 3 4 4	27         2421         2410         2406         2398         2392         2392         2397         1         1         2         2         3         3           70         2364         2388         2335         2335         2335         2335         2337         1         1         2         2         3         3           16         2364         2284         2283         2283         2283         2283         2         3 </td <td>22 225/ 2252 2247 2242 2237 2232 2227 1 1 2 2 3 3 12 2 2 3 2 5 2 5 2 5 2 5 2 5 2 5 2 5 2 5</td> <td>19         2114         2110         2105         2101         2095         2088         0         1         1         2         3         3           15         2070         2066         2053         2053         2045         0         1         1         2         2         3         3           15         2070         2066         2053         2053         2045         0         1         1         2         2         3</td> <td>32 1988 1984 1980 1976 1972 1969 1965 0 1 1 2 2 2</td> <td>33         1946         1941         1938         1934         1934         1934         1331         1927         0         1         1         2         2           16         1912         1905         1901         1888         1834         1890         0         1         1         1         2         2           10         1875         1806         1865         1865         1865         1855         0         1         1         1         2         2           10         1877         1869         1865         1865         1865         1         1         1         2         2</td> <td>45 1842 1838 1835 1832 1828 1825 1821 0 1 1 1 2 2 2</td> <td>3 4 5 6 7 8 9 1 2 3 4 5 6</td>	22 225/ 2252 2247 2242 2237 2232 2227 1 1 2 2 3 3 12 2 2 3 2 5 2 5 2 5 2 5 2 5 2 5 2 5 2 5	19         2114         2110      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x         0           10         100         1000           11         9091         111           12         83333         114           13         8266         870           14         7143         8665           15         6557         8333           19         5556         8000           20         5000         5000           21         4762         846           23         3333         3448           22         4467         3704           23         3333         3333           30         3333         3333           30         3333         3333           30         3333         3333           30         3333         3333           30         3333         3333           30         3333         3333           31         2278         3448           2778         3333         3333           31         2266         2841           31         2273         284           31         23256         3448           32335         3448         2841		2 3 4 5 6 7 8 9 1 2 3	1 9804 9709 9615 9524 9434 9346 9259 9174 9 18 28 37 4	9         892.9         8850         8772'         8696         8547         8475         8403         8         15         23         31         38           4         8197         8193         8065         8000         7837         7834         7813         7752         7         13         20         23         31         38           4         8197         8197         8137         7814         7813         7752         7         13         20         25         33           4         7516         7463         7407         7353         7299         7246         7194         6         11         17         22         28	<b>2. 7</b> 042 6993 6944 6897 6849 6803 5757 5711 5 10 14 19 24 2 3 6579 6506 6494 6897 6449 6803 6525 6573 6711 5 10 14 19 24 2 1 6173 6579 6506 6494 6452 4540 5389 6525 5573 4 7 11 15 18 2 1 6173 6098 6051 6024 5588 5952 5977	5         5431         5730         5774         5682         5650         5618         5587         3         7         10         13         16         20           5         5435         5445         5435         5405         5376         5348         5319         5291         3         6         9         12         17         13         15         15         15         5495         5445         5426         5376         5348         5319         5291         3         6         9         12         17         15         15         15         16         12         17         16         16         16         16         16         16         16         16         16         16         16         16	5 4950 4928 4902 4878 4854 4831 4808 4785 2 5 7 10 12 14	4717         4655         4653         4653         4551         4530         4508         4587         4565         2         4         7         9         11           5         4505         4384         4425         4405         4386         4367         2         4         6         8         10         12           1         4510         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3236 1 2 3 4 5 6	5         3205         31555         3155         3155         3	2224         2915         2807         2892         2892         2892         2894         2865         1         2         3         3         4         5         6           7         2841         2892         2892         2892         2892         2865         1         2         3         3         4         5         6         7         2841         2892 </td <td>2 2688         2687         2660         2653         2646         2653         2646         2653         2644         4         4           2 2618         2611         2604         2537         2571         1         1         2         3         4         4           2 551         2551         2584         2577         2571         1         1         2         3         4         4           2 551         2551         2584         2577         2571         1         1         2         3         4         4</td> <td>1 2488 2481 2475 2469 2463 2457 2451 2445 1 1 2 2 3 4 4</td> <td>8 2427 2421 2415 2410 2404 2398 2392 2397 1 1 2 2 3 3 5 5 2370 2364 2358 2353 2347 2342 2336 2331 1 1 2 2 3 3 1 2315 2309 2304 2299 2294 2288 2283 2278 1 1 2 2 3 3</td> <td>2 2662 2267 2252 2247 2242 2237 2232 2227 1 1 2 2 3 3 2 2212 2208 2203 2198 2199 0 1 1 2 2 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2</td> <td>2119         2114         2110         2105         2101         2096         2095         2088         0         1         1         2         2         3         3           1         2075         2076         2053         2058         2053         2049         2045         0         1         1         2         2         3         3           2         2033         2058         2053         2048         0         1         1         2         2         3</td> <td>i 1992 1988 1984 1980 1976 1972 1969 1965 0 1 1 2 2 2</td> <td>1953         1948         1934         1934         1931         1927         0         1         1         2         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x 2 111 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2		1 2 3 4 5 6 7 8 9 1 2 3	9901 9804 9709 9615 9524 9434 9346 9269 9174 9 18 28 37 4	9009         8929         8850         8772'         8696         8571         8547         8475         8403         8         15         23         31         38           8264         8137         8130         8065         8000         7937         7814         7813         7752         7         13         20         25         33           7634         7515         7519         7463         7407         7353         7299         7246         7101         17         22         28	7092_7042 6933 6944 6897 6849 6803 6757 6711 5 10 14 19 24 2 6523 6579 6536 6494 6897 2410 2595 6529 5429 4 8 13 17 21 2 -6211 6173 6139 (596 6061 6024 5688 5952 5917 4 7 11 15 18 2	5848         5814         5730         5747         5822         5650         5618         5587         3         7         10         13         16         20           5525         5456         5405       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2257 2267 2247 2232 2237 2232 2227 1 1 2 2 3 3 2217 2212 2208 2293 2198 2179 0 1 1 2 2 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	2123         2119         2114         2110         2105         2101         2095         2095         2088         0         1         1         2         2         3         2           2075         2075         2076         2056         2058         2053         2043         2045         0         1         1         2         2         3         2           2037         2037         2038         2015         2016         2011         2018         2004         0         1         1         2         2         3         2	1996 1992 1988 1984 1980 1976 1972 1969 1965 0 1 1 2 2 2	1957         1957         1957         1957         1957         1957         1957         1957         1957         1951         1928         1931         1927         0         1         1         2         2           1919         1912         1908         1905         1901         1898    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	10800 8124 0754 8 10 8 10 8 10 8 10 8 10 8 10 8 10 8 1	0 1 2 3 4 5 6 7 8 9 1 2 3	1000 3901 3804 3709 9615 9524 9434 9346 9269 9174 9 18 28 37 4	9091         9009         8929         8850         8772         8696         8547         8475         8403         8         15         23         31         38           8333         8264         8197         8130         8065         8000         7837         7814         7813         7752         7         13         20         26         33           7834         7814         7814         7813         7792         7         13         20         26         33           7832         7519         7463         7407         7353         7299         7246         7194         6         11         17         22         28	7143         7092_*         7042         6933         6844         6897         6849         6803         6757         6711         5         10         14         19         24         24         26         24         26         24         26         24         26         24         26         27         27         21         15         18         27         12	5882.         5848         5814         5780         5747         5714         5680         5610         5618         5587         3         7         10         13         16         20           5556         5525         5495         5445         5376         5348         5319         5291         3         6         9         12         17         17         16         16         16         5683         5548         5348         5348         5319         5291         3         6         9         12         17         5         5         16         17         16         5783         5538         5328         5328         5329         5038	5000 4975 4950 4925 4902 4878 4854 4831 4808 4785 2 5 7 10 12 14	4762         4733         4717         4656         4551         4550         4660         4587         4566         2         4         7         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     3         4         5         6           2778         2770         2782         2812         2890         2892         2986         1         2         3         4         5         6           2778         2770         2782         2817         2809         2804         2841         2832         3         4         5         6         7         7         4         5         7         7         7         7         7         4         5         7         7         7         7         7         7         5         7         7         7         5         7         7         7         5         7         7         7         5         7         7         7         5         7         7         7         5         7         7         7         5         7         7         7         7         7         7         7         7         7         7         7         7         7         7         7<td>Z103         2695         2688         2657         2660         2653         2646         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2320 2315 2309 2304 2299 2294 2288 2283 2278 1 1 2 2 3 3 3 2356 2357 2369 2394 2288 2283 2278 1 1 2 2 2 3 3 3 2356 2357 2369 2394 2388 2283 2278 1 1 2 2 2 3 3 3 2356 2356 2356 2356 2356 23</td><td>2273 2268 265 2267 2255 2247 2242 2237 2232 2227 1 1 2 2 3 3 2 2 2 2 2 2 2 2 2 2 2 2 2 2</td><td>2128         2123         2119         2110         2105         2101         2095         2088         0         1         1         2         3<td>2000 1996 1992 1988 1984 1980 1976 1972 1969 1965 0 1 1 2 2 2</td><td>1961         1957         1963         1949         1946         1941         1938         1934         1931         1927         0         1         1         2         2           1923         1919         1916         1912         1906         1901         1888         1894         1834         1931         1927         0         1         1         2         2           1923         1919         1916         1901         1888         1894         1894         1894         1930   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1976 1972 1969 1965 0 1 1 2 2 2</td><td>1961         1957         1963         1949         1946         1941         1938         1934         1931         1927         0         1         1         2         2           1923         1919         1916         1912         1906         1901         1888         1894         1834         1931         1927         0         1         1         2         2           1923         1919         1916         1901         1888         1894         1894         1894         1930         0         1         1         2         2           1887         1883         1880         1876         1869         1862         1855         0         1         1         2         2</td><td>1852 1848 1845 1842 1838 1835 1832 1828 1825 1821 0 1 1 1 2 2 2</td><td>0 1 2 3 4 5 6 7 8 9 1 2 3 4 5 6</td></td>	Z103         2695         2688         2657         2660         2653         2646         2663         1         2         3         4         4           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